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ABSTRACT

Presented is an account of a day-and-a-half seminar which was held for selected mathematics educators from six states. It was sponsored by the Regional Exchange (RX), a National Institute of Education (NIE) dissemination project headquartered at the Southwest Educational Development Laboratory (SEDL). The seminar had two important purposes: (1) to foster an exchange of information and resources between mathematics researchers and mathematics practitioners; and (2) to lend support to the mathematics preservice/inservice activities in the six-state region. Two researchers were on the program. One was a mathematics educator who discussed junior high school mathematics teaching in terms of diagnosis, remediation, mathematics anxiety, and problem solving. The other was a research scientist with the Classroom Organization for Effective Teaching (COET) who discussed classroom management research in the junior high school classroom. A synthesis of each of these discussions is given. Questions and answers which occurred during the discussions have been incorporated into the text and hand-outs given during the discussions are included following the texts. Also included are written questions which the participants presented to the researchers, and the answers and recommended resources with which the researchers responded. (Author/MK)

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SEDL/REGIONAL EXCHANGE

R&D SPEAKS
IN
JUNIOR HIGH SCHOOL MATHEMATICS:

A Research Seminar for Practitioners

SOUTHWEST EDUCATIONAL DEVELOPMENT LABORATORY

AUSTIN, TEXAS

U S DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

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Austin, Texas

PROJECT STAFF: Preston C. Kronkosky (Director), Sharon Adams, Jan Schechter, Karen Olsen, Nancy Baker Jones, Morrie Schulman, Jack Lumbley, Barbara Dupree, Teri Aleman

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FOREWORD

The SEDL Regional Exchange (SEDL/RX) Project has provided information and technical assistance services since October 1976 to educators in five states: Arkansas, Louisiana, New Mexico, Oklahoma, Texas and, since April 1979, to educators in Mississippi as well. It is one of seven Regional Exchanges in the nation-wide Research & Development Exchange (RDx), funded by the National Institute of Education, which lists as a major goal the dissemination of information about educational research and development (R&D). To assist in accomplishing this goal, the SEDL/RX staff designed and sponsored the R&D Speaks in Junior High School Mathematics seminar in Dallas, Texas on August 8 and 9, 1979. In an effort to record and pass on to others some of the experience and knowledge that was shared during that day and a half meeting, this document, R&D Speaks in Junior High School Mathematics: A Research Seminar for Practitioners, was compiled.

This is the second of a series of such "R&D Speaks" conferences on various topics which will be sponsored by the SEDL/RX during the current year. The first was the R&D Speaks in Reading conference held in Dallas, Texas on November 16 and 17, 1978. These conferences will provide opportunities for sharing, communication, and growth among researchers, clients, and users of the knowledge and products of educational research and development.

James H. Perry
Executive Director
September 1979

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INTRODUCTION

This document, R&D Speaks in Junior High School Mathematics: A Research Seminar for Practitioners, is the account of a day and a half seminar which was held August 8-9, 1979 in Dallas, Texas for selected mathematics educators from Arkansas, Louisiana, Mississippi, New Mexico, Oklahoma, and Texas. Sponsored by the Regional Exchange (RX), a National Institute of Education (NIE)-funded dissemination project headquartered at the Southwest Educational Development Laboratory (SEDL) in Austin, Texas, the seminar had two important purposes:

- to foster an exchange of information and resources between mathematics researchers and mathematics practitioners;
- to lend support to the mathematics preservice/inservice activities in the six-state region.

Why junior high school mathematics?

Beginning only with the promise to bring mathematics researchers together with mathematics practitioners, the SEDL/RX approached its Advisory Board with a request for the names of mathematics educators in their states who might advise us regarding the most useful focus for the meeting. Each Advisory Board member named a mathematics specialist in the state education agency (SEA)--in each case, a person whose job required service work in the field. These specialists were then contacted by letter with an explanation of the "R&D Speaks" concept--researcher/practitioner exchange--and a list of possible mathematics topics (See Appendix A). Our contacts were asked to designate both preferred topics and preferred grade levels. Approximately ten days later, pre-arranged phone calls were placed to the contacts to discuss their preferences.

The idea of interacting with researchers appealed to most of the specialists, although they were very clearly cautious. "Will they (the researchers) be practical?" "Will only theory be discussed?" "Will these researchers understand what we want?" Somewhat bravely, we assured them that the researchers would indeed specifically address their needs though we, at the time, knew neither the topics nor the researchers we were so confident about. Time and again the specialists expressed the desire to "sit and talk to the researchers" and "ask questions." "We don't want to play games or have cute little activities," one told us, "we don't need a product display; we want to hear the information 'straight.'"

And what did they want to hear about? Junior high school mathematics, unanimously. And, in particular, they wished us to address diagnosis, remediation, problem solving, and math anxiety. Both diagnosis and problem solving received a unanimous response while remediation and math anxiety were endorsed by five of the six respondents. Through the course of all the conversations, though, there appeared an additional concern which had not been on the list of topics. It would sometimes be expressed in this way: "The junior high school student poses so many discipline problems. How do we get the student to sit still long enough to benefit from remediation?" or this way: "How can a teacher manage classroom of thirty or more junior high students and diagnose them individually at the same time?" Clearly there was concern about classroom management techniques with junior high school students, although no one used those words.

Several of the specialists reminded us of the opportune timing of the proposed meeting. Information and ideas encountered in early August could still influence fall inservice.

The Researchers

It was clear that we needed to find researchers well-grounded in their fields, able to synthesize findings, and who, at the same time, were sensitive to the requirements of inservice work. We were most fortunate to find two such researchers interested in working with practitioners.

Dr. Marilyn Suydam agreed to approach junior high school mathematics teaching in terms of diagnosis, remediation, math anxiety, and problem solving. Dr. Suydam is Adjunct Professor, Mathematics Education, and Faculty Research Associate, ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, The Ohio State University. She has done a number of mathematics research syntheses and currently serves as a consultant on a major mathematics (K-6) synthesis project for the Research and Development Interpretation Service (RDIS), a national contractor in the Research and Development Exchange (RDx) and housed at CEMREL, Inc. in St. Louis, Missouri.

Dr. Edmund E. Emmer, Research Scientist with the Classroom Organization for Effective Teaching (COET) project at the R&D Center for Teacher Education at The University of Texas at Austin, agreed to discuss classroom management research in the junior high school classroom. Dr. Emmer has participated in two junior high studies with the COET project, and additionally, still teaches junior high school mathematics part-time. He has also had extensive inservice experience.

The Practitioners

"Practitioner" is an interesting word. Sometimes it is used as a euphemism for "teacher," but in our case, we are referring to any educator with the opportunity to apply basic research findings to classroom operation.

Participants in the "R&D Speaks in Junior High School Mathematics," therefore included teachers, principals, state education agency mathematics specialists and coordinators, university mathematics professors, and university teacher educators from the SEDL/RX region: Arkansas, Louisiana, Mississippi, New Mexico, Oklahoma, and Texas.

The Format

The seminar format was chosen for the meeting because it seemed to offer the participants the best opportunity to hear the researchers' comments, and then to comment and question as appropriate. Accordingly, researchers were asked to make initial presentations and schedule time for questions. There was also an opportunity for written questions to be submitted to the researchers, and time scheduled for the various state representatives to discuss inservice implications.

Additional support

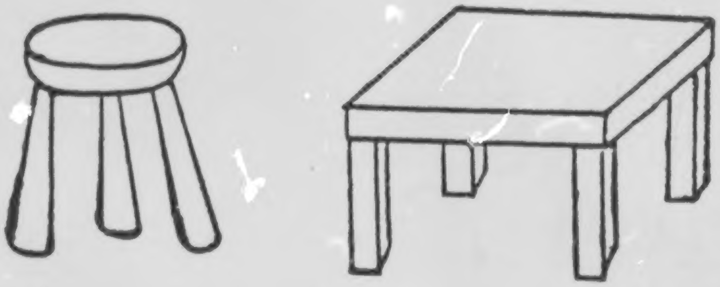
Planners of the "R&D Speaks in Junior High School Mathematics" benefited greatly from the experiences and advice of Sharon Adams, Coordinator of SEDL/RX's "R&D Speaks in Reading." In addition, Sandra Orletsky, Assistant Director of the Regional Exchange at the Appalachia Educational Laboratory (AEL/RX) and Jane Williams, Intern with AEL/RX, attended the seminar and provided valuable feedback through an objective viewpoint of the proceedings. Underlying all of this support were the efforts of Linda Reed, Director of the Research and Development Interpretation Service (RDIS), and Mark Driscoll, Coordinator of RDIS' mathematics synthesis project, who provided access to several material resources and also sponsored an RDx Planning Meeting which introduced the Exchanges to several mathematics researchers, among them, Dr. Marilyn Suydam. A bibliography of the materials supplied to SEDL/RX by RDIS is included in Appendix B.

The "R&D Speaks in Junior High School Mathematics" was planned, coordinated, and documented by SEDL/RX Dissemination Specialist Janis O'Driscoll Schechter, with the assistance of Nancy Baker Jones, Technical Writer. The advice and participation of Dr. Preston C. Kronkosky, Director of SEDL's Division of Field Services and Dissemination, was greatly valued. Dr. Kronkosky, who is also the Director of the SEDL/RX,is a former mathematics teacher and coordinator, and so brought an important perspective to the panel discussion. Evaluation of the conference was carried out by Morrie Schulman, SEDL/RX Research Associate. Conference details such as room reservations and reimbursement, and complete preparation of this document for distribution were in the very capable hands of Teri Aleman, Senior Secretary for the Regional Exchange.

A Word About Junior High School Mathematics Research

The preceding description does not reflect the apprehension we felt when junior high school mathematics was unanimously chosen as our seminar topic. Research has been done on elementary mathematics, but much less has been done on junior high mathematics. Even less has been attempted at the secondary level. But with help of Dr. Suydam and Dr. Emmer, we began to see that though the research has only begun, there are findings that can be applied to the classroom, most notably in problem solving and classroom management. There is a need, therefore, for a continuing dialogue between researchers and practitioners, and SEDL/RX hopes to provide opportunities for that dialogue whenever possible.

We offer here a summary of the presentations and of the issues discussed during this first mathematics seminar. And just in case you have forgotten what junior high mathematics is like, we've included a problem for you to solve (courtesy of Marilyn Suydam).



Lynn's dad worked in a carpenter shop. During the day he made only 3-legged stools and 4-legged tables. At the end of the day he had used 31 legs. How many stools and how many tables did he make?

Now that you're prepared, please take a look at what research has to say about junior high school mathematics.

REGIONAL EXCHANGE

AGENDA

for

"R&D SPEAKS IN MATHEMATICS"

Wednesday, August 8, 1979

8:30 - 9:00am	Coffee, juice, pastries
9:00 - 9:15am	Introductions Opening Remarks
9:15 -11:30am (includes a break)	Junior High Mathematics: Considering the Research Dr. Marilyn Suydam, ERIC/SMEAC Dr. Suydam will discuss the research, its implications for inservice, and respond to questions and comments from the participants. The areas of focus will be diagnosis, remediation, problem solving, and math anxiety.
11:45 - 1:00pm	LUNCH
1:00 - 3:30pm (includes a break)	Classroom Management and the Junior High Classroom: Considering the Research Dr. Ed Emmer, UT R&D Dr. Emmer will discuss the research, its implication for inservice, and respond to questions and comments from the participants.
3:30 - 3:40pm	Participant Reaction Participants will be asked to write down questions or comments for one or both of the researchers to respond to during the first session on Thursday.

(over)

Thursday, August 9, 1979

8:30 - 9:00am Coffee, juice, pastries

9:00 -10:00am Researchers on the Firing Line

The researchers will respond to questions and concerns of the participants. The panel of researchers will be moderated by Preston C. Kronkosky, Director, SEDL Division of Field Services and Dissemination. The panel will respond to:

. participants' written comments and questions;

. Dr. Kronkosky's comments and questions;

. participants' comments and questions during the discussion.

10:00 -10:45am Implications for Inservice

Participants will gather in state groups to discuss the ways that mathematics research and classroom management research can be used in inservice.

10:45-11:00am BREAK

11:00-11:30am Report of State Groups

11:30-11:45am Issues for Future Research

The conference will have addressed the research that has been or is being conducted. From the view of the practitioner, participants will have the opportunity to talk about those issues which they would like to see research address.

11:45-12:00 noon
Closing Remarks
Evaluation
Reimbursement Forms

PARTICIPANT LIST FOR R&D SPEAKS IN MATHEMATICS

Arkansas	Dr. Charles Watson
	Dr. James Fulmer
	Mrs. Helen McKinney
Louisiana	Dr. Jean Reddy
	Ms. Bonnie Ross
	Dr. Jack Garon
Mississippi	Mrs. Dollie Mosley
New Mexico	Mr. Bill Trujillo
	Mr. Bev Graham
Oklahoma	Dr. Joe Bob Weaver
	Mrs. Alice Jackson
	Mrs. Carolyn Nicks
	Mr. Ed Jackson
Texas	Dr. Alice Kidd
	Mr. Bill Scannell
	Mr. Bill Hopkins
	Dr. Ray Carry
SEDL/RX	Dr. Preston C. Kronkosky
	Ms. Jan Schechter
	Ms. Nancy Baker Jones
Consultants	Dr. Marilyn Suydam ERIC Clearinghouse (ERIC/SMEAC) The Ohio State University
	Dr. Ed Emmer R&D Center for Teacher Education University of Texas at Austin

DR. MARILYN SUYDAM:

THE RESEARCH IN JUNIOR HIGH SCHOOL MATHEMATICS

- Math Anxiety
- Diagnosis
- Remediation
- Problem-Solving

The following pages comprise a synthesis of Dr. Suydam's discussion at the R&D Speaks in Junior High School Mathematics seminar. Questions and answers which occurred during the discussion have been incorporated into the text, and hand-outs given during the discussion are included following the text.

The purpose of the "R&D Speaks in Mathematics" seminar is to put state education agency (SEA) and local education agency (LEA) people in contact with recent mathematics research and researchers. The assumption is that research can assist the practitioner--and it can--if expectations are realistic. "Research doesn't have clear answers." Research findings don't offer the one final solution, but research implications do indicate some of the most effective directions for practitioners to pursue. Some aspects of research have clearer implications than others because of the amount of investigation so far undertaken and the development of appropriate methodologies. Research in any area of junior high school mathematics, for example, is only beginning to provide guidance to classroom teachers because researchers have heretofore focused on elementary mathematics, and comparatively little has been specific to the junior high level.

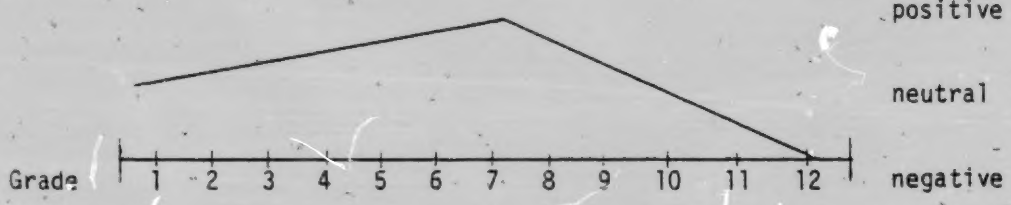
The topics to be considered at this seminar are logical ones--math anxiety, diagnosis, remediation, and problem-solving. They are all clear concerns of the junior high mathematics teacher. Research can address these concerns with varying degrees of specificity; little has been done in math anxiety, somewhat more has been done in diagnosis and remediation, and extensive research continues in problem-solving. These topics will be treated in this order, beginning with math anxiety and ending with problem solving.

Math Anxiety

Little research has been done on math anxiety, although much has been written about it. Perhaps the most well-known work is Sheila Tobias' on Overcoming Math Anxiety. Tobias' book is really a collection of personal perceptions, rather than research, but it is widely read. Actually, what research there is indicates that there is a low correlation between attitudes and achievement--in other words, a student's attitude toward mathematics would seem to have little effect on that student's performance in mathematics. But most of us find that very difficult to believe, in spite of the research, and the problem may well lie with the measurement instruments. How do we measure attitudes? What instruments are appropriate for such a task?

Positive Attitudes toward mathematics begin to decline in junior high school.

Well, if we can't yet measure the existence and effects of math anxiety, can we say anything about attitudes toward mathematics? A little. It seems that, generally, attitudes toward mathematics reflect this pattern:



Attitudes toward mathematics in elementary school begin neutral or high, continue rising through 4th-6th grade, and then decline in 7th grade, never to rise again. Why? Researchers and practitioners both speculate about two possibilities:

- peer pressure
- our cultural values regarding mathematics

Most of us readily accept the increased influence of peers on the 12 to 14-year-old (although, as we shall see momentarily, adults still have influence, too), and the student of this age finds the acceptance and understanding of social values to be of paramount concern. But have we thought about the assumptions that underlie our cultural values regarding mathematics, and how these affect the young teenager?

For instance, an issue given more and more consideration is that of the relationship of the student's sex to the student's anxiety about mathematics. The little data that exist in this area indicate that there is not much difference in math anxiety between male and female students, yet we know that there are far fewer females enrolled in advanced mathematics classes than males. Current research activities now underway may produce findings which will be useful in analyzing this situation, and some of these findings may also shed light on a basic Western assumption made about mathematics.

Mathematics: The Latin for twentieth-century education?

Even a cursory (ə†!!!!†əə) look at our values regarding mathematics reveals certain specific "givens":

- in mathematics, there is clear right and wrong;
- in mathematics, there is a correct way to find the answer;
- in mathematics, formulas show the pre-set procedure--just memorize the formulas.

There is little or no talk of using common sense in mathematics, much less creativity. Ask the average person a simple mathematical question, and chances are s/he will try to write out an algorithm on the "blackboard of her/his mind." For instance, if a man was 38 in 1976, in what year was he born? How can the answer be found? One could subtract 38 from 1976. But one could also:

- subtract 40 from 1976 and then add 2 to the result;
- think of someone who was 38 in 1976 and remember the person's birthdate;
- compare the man's age to one's own, and compute the difference.

There is more than one way to find the answer, but the common belief that in mathematics there's only one set procedure contributes to the anxiety many people feel about mathematics. Some math teachers and researchers think that calculators might relieve some of the anxiety in "figuring out" answers, but others object because "it makes things too easy." Thus, it might be said that mathematics has replaced Latin in modern schools because it can be difficult, require rote learning, and discipline the mind (the supreme challenge to the aspiring scholar). It is also interesting to note that young men predominantly studied Latin, and it is young men that predominantly study mathematics.

Anxiety differs according to ethnic group

Researchers are detecting clearer correlation, though, between ethnicity and math anxiety. It was mentioned earlier that the view of mathematics as primarily computation skills or a series of algorithms reflects a Western perspective. Indeed, people from Western Europe and the United States seem to have this uneasy truce with math, while people from Asia have a higher interest in mathematics study. Within the United States, the problem intensifies. Blacks seem to experience more math anxiety than Whites, and the Native American population, particularly in the Southwest, suffers from it most of all. To date, no known research has been done regarding Math anxiety in Hispanic populations.

Many questions, but the solutions are still being sought

Unfortunately, at this early stage in math anxiety research, no remedies can be proposed. Some research suggests that students' anxiety can be reduced somewhat by clearly communicating to the student what is to be learned. Making the learning task clearer seems to reduce the mystery of the mathematician's "secret formulae."

But this is not to say that establishing minimum standards will reduce math anxiety. The pressure of passing *a* test in a subject essential to many occupations may just increase the pressure. Further, what kind of minimum standards are being set? If the standards continue to give primary importance to the knowledge base and set procedures, they will have only proved to students that in mathematics classes "teachers will tell them what to do," and then to succeed in class, "students have to *recall* that information." In setting minimum standards, mathematics educators must ask, "What else do we [adults] need besides computation?" And in asking that question, educators need to be mindful of the attitudes of parents, and the public, in general. It is not uncommon, after all, to hear people comment on their poor grades in school or boast about their ignorance in mathematics--"I can't even balance my checkbook." It is acceptable to have poorly developed math skills, but people, on the average, do not say, "I can't even read the newspaper." Reading is different, somehow. How much of the decline in positive math attitudes of junior high students can be traced to the values that adults around them express? If this is a significant influence, what can be done?

At this point, the math anxiety of junior high school students is matched only by the anxiety of math researchers and educators who are trying to find ways to help.

Diagnosis

Diagnosis and remediation are of great importance to junior high school mathematics teachers for their students are often seen as "being in transition." "Make sure they've got what they were supposed to get in K-6 because next year they'll be getting *high school* math." By seventh and eighth grades, the teacher is confronted with a variety of developed and partially-developed skills in each class of 25-40 students, with 45-60 minutes/day to meet their needs. Mathematics research indicates no fail-safe teaching techniques for these kinds of classes, but does indicate that the "teacher is the key variable." Enthusiasm, support, and mathematics knowledge is important, and the need for appreciating the positive is carried over to diagnosis.

Diagnosis identifies strengths as well as weaknesses

In thinking of junior high students as being in transition, it is tempting to worry about those who need to "catch up" in order to reach the next plateau. But there are students who are far beyond their contemporaries, with much more developed skills, and with just as many needs to be addressed. Little research has focused on the needs of the gifted and talented, and only very recently have funding sources begun to underwrite needed studies to help our brightest pupils. Unfortunately, like all the others I speak of, I, too, will direct most of my comments toward the weaknesses diagnosis identifies.

Mastery takes time

Diagnosis of junior high math students reveals weaknesses in performing operations that "should have been mastered in grade school." But review the weaknesses and one finds that most students' achievement in whole number computation conforms to the following pattern, in descending order of mastery: addition, subtraction, multiplication, division--exactly the order in which they are taught. Students have been adding longer than they have been dividing, and division is a more difficult skill. Weaknesses in seventh and eighth grade may often indicate the need for more practice time so that mastery can be achieved.

Studies have shown that the three most common troublespots are:

- division
- subtraction with regrouping
- fractions

Error analysis

Whatever the problem, diagnosis of it begins with error analysis--where are the students making mistakes? Error analysis is especially important because the alternative is to follow the "so-called national curriculum,' the textbook. Error analysis allows the teacher to choose to use the appropriate parts of the curriculum at the appropriate time. This type of diagnosis may be carried out in several ways:

- interviews
  An interview can take 5-30 minutes. The teacher may ask the student to explain how to solve a particular problem, or the teacher may ask the student to describe the process as the student is carrying it out.

- standardized tests
  Too often these tests are not used as a diagnostic aid. One could select sample problems and analyze students' success with each. Or one could look at the test papers of a few who did well and a few who did poorly, and compare. Access to computer analysis would provide the opportunity to study each question or each paper.

- teacher-made tests
  Require students to show work and then compare patterns of successful and unsuccessful students.

- daily work
  Focus on the operation of a particular strategy or problem for the analysis.

- observation of behaviors

- analysis of records

These methods are significant in that they make use of data already available to the classroom teacher. An important consideration, as it is the belief of some lawyers that schools which have data on students and do not use it to benefit students (for whatever reason) can be liable for legal action by students or parents. A teacher should not construe, however, that one or more of these methods must be used with each student during the same time period. Select students for diagnosis according to their difficulties; it is possible to diagnose two students per week without spending a large amount of time, yet gathering useful data.

Remediation

Diagnosis is vital for remediation, whether remediation takes the form of a complete, specially developed program, or is conducted through a variety of techniques organized for a particular class. Studies are looking at both of these methods and some suggestions can be made.

Drill and practice

For some, diagnosis results in the development of special programs. Little now is known about what makes a special program work, other than it should be directed toward very specific objectives. Since such focused attention will likely increase drill and practice, studies show that "if you design a focused program, it will probably help." However, the research also indicates that spending at least 50% time on developmental activities--and therefore less than 50% of the time on practice--will result in higher achievement.

Review must be planned. The teacher should expect both understanding and competence, and that, too, means drill and practice. Often there is no overt provision for review, but to be effective, it must be systematically planned, and it must be continuous.

Manipulatives and materials

The use of manipulatives/materials with junior high students is a matter of some discussion among teachers. The math lab, organized activity-based learning, has been one of the few places where manipulatives have been "acceptable," but generally, most math labs have focused on non-standard topics. We don't know, then, about the usefulness of math labs in remediation, but we can say something about the usefulness of manipulatives/materials. Studies indicate that materials promote student achievement in 50% of the cases, have a negative effect in very few cases, and have no significant difference in the remaining cases. This means that materials can't hurt, and they probably could help. "But that's baby stuff--the students won't use them." It has been shown that students' balkiness depends on the teacher. If the teacher approaches the lesson with a positive attitude, and demonstrates that the manipulatives will help the student learn, then the junior high school student will use them. Manipulatives and materials are strongly advised for this age group.

Tutors

The research on the effectiveness of tutors is variable because tutoring can take so many forms. Peer tutoring appears to help both the student-learner and the student-teacher, and for this reason, there has been some success in using slower learners as tutors. The use of adults as tutors mandates training. Adults tend to depend on their backgrounds, and when a student encounters difficulties, many adults will respond, "do it over again." Adult tutors need training to provide alternative strategies to a student, when one strategy doesn't seem to work.

Other advice for effective remediation

Research indicates that there are several other ways to promote effective remediation, and among these are:

- attention
  Praise helps students achieve, but so does criticism. One should avoid extremes in either direction, but provide students with appropriate recognition. It would seem, however, that even criticizing is better than ignoring.

- enhance students' self concepts
  Research indicates no significant difference as to the effects of self concept on math achievement, but most researchers and educators feel that there must be a relationship.

- calculators
  Slow learners appear to be helped by calculators, but teachers need help in designing effective ways of using calculators so that students think about the process they are practicing.

- records
  Keep careful records so you can diagnose problems specifically. Don't do general remediation, such as repeating 5th grade curriculum.

- involve the student in the diagnosis process and spell out short term goals for accomplishment

- applicability
  Plan experiences related to the student's present and future needs.

- slow the pace of initial instruction

- specify the goals of your remediation program

- give maximum explanation and guidance during lessons

- use games and a variety of activities at the appropriate level

Underlying all of these suggestions are two important "musts" for remediation. First, remember that different methods will be required for different pupils. And second, teachers must expect achievement.

Problem-Solving

We are running out of time, but I do want to mention some resources in problem-solving--the most important of the basic skills:

- The 1980 National Council of Teachers of Mathematics Yearbook (Stephen Krulik, editor) will focus on problem-solving. It will address both the nature of problem solving and the implications for teaching problem-solving strategies. The Yearbook, for instance, indicates some characteristics of a good problem solver:

  - analyzes concepts and terms;
  - recognizes likenesses and differences;
  - focuses on the structure of the problem, not on the details.

- The key to problem solving, of course, is the strategy. The state of Maryland has incorporated the following analysis of strategies in its scope and sequence: (see next page)

- For the Ohio Regional Conferences, we developed a resource packet on problem solving.

  In it you will find a collection of different kinds of problems, rather than merely the exercises you're likely to find in textbooks.

- In the fall, the ERIC/SMEAC Clearinghouse will have available a document on problem solving edited by Goldin and McClintock. This publication will record research findings on problem solving and include lesson plans.

- Problem solving using calculators is addressed in another document available from ERIC, Calculator Cookery. It outlines the prior knowledge needed and lesson objectives as an introduction to each worksheet.

- Soon to be published is Steven Meiring's book, Problem Solving: A Basic Mathematics Skill. Columbus: Ohio Department of Education (to be published in 1979).

PROBLEM SOLVING SECTION FROM MARYLAND MATHEMATICS SCOPE AND SEQUENCE

GOAL IV. TO SOLVE SPECIFIC MATHEMATICAL PROBLEMS.		
GRADE	ANALYZE PROBLEM SITUATION & DEVELOP LOGICAL SOLUTION SEQUENCE	SOLVE & CHECK
1	*Interpreting picture problems*	*Solving informally*
2	*Stating given information* *Stating what is to be found* *Acting out problems keeping a record of steps*	*Acting out*
3	*Illustrating problem with a diagram or sketch* *Restating problem in own words* *Choosing correct operation(s)*	*Using trial and error*
4	*Stating information needed* *Sorting and classifying data*	*Estimating result and testing in problem* *Performing mathematical manipulations* *Testing solution by checking manipulations*
5	*Identifying extraneous information* *Replacing large numbers by small ones to determine method of solution*	*Testing reasonableness of solution*
6	*Classifying problems* *Choosing appropriate formula, if one is required* *Writing flow charts to consider all alternatives*	*Determining the replacement set for each unknown* *Checking solution to see if it satisfies all conditions*
7	*Making model of problem, if required* *Selecting a variable to represent unknown quantity* *Translating problem into mathematical sentence*	*Solving mathematical sentences* *Finding counterexamples* *Rounding off numerical solution, if required*
8		*Drawing conclusions or generalizations* *Determining uniqueness of solution*
9 - 12	*Determining required precision of solution* *Defining problem* *Identifying possible solutions* *Hypothesizing* *Gathering information* *Interpreting information* *Looking for patterns* *Generalizing* *Determining necessary and/or sufficient conditions* *Drawing conclusions*	*Checking for misuse of data*

Suydam, Marilyn N. Untangling Clues from Research on Problem Solving. In 1980 NCTM Yearbook on Problem Solving (Stephen Krulik, editor). Reston, Virginia: National Council of Teachers of Mathematics. To be published April 1980.

CLUES FOR TEACHING

Let's consider these points one by one, and note just one experience that a child could have to promote facility on that point (you can think of many others):

a. Use a term (such as "equals") in a way that makes clear first its mathematical sense, and then its use in life outside the school.

b. Classify problems (they'll probably select "addition" or "baseball" as problem types; when given all "baseball" problems, what other comparisons can be made?)

c. Cite only the aspects of a problem without which there would be no problem.

d. "Cross out" irrelevancies in a word, picture, oral, or other form of a problem.

e. Estimate answers and analyze what route is taken to attain the estimate.

f. Describe spatial and numerical ideas not only in words, but also by drawing pictures, by using physical materials, by building models.

g. Guess what rule or function might apply to a sequence of numbers after exploring a few examples -- then verify the guess.

h. Use of a variety of methods implies that the child knows a variety of methods: we'll talk more about this.

i. Praise still works wonders.

j. Involve the child in using tests as a way for the child (and not just the teacher) to evaluate learning.

CLUE FOR TEACHING

How can we help children focus less on details? One thing which may help: don't focus all of their attention on those details.

CLUE FOR TEACHING

Teach children a variety of different strategies which they can apply in various problem-solving situations, plus an overall plan for how to go about problem solving. More about this later!

CLUE FOR TEACHING

Allow children sufficient time to solve problems. All problems cannot be solved within the confines of one mathematics lesson; some need to be "mulled over" for longer periods of time. Of course, some problems can be solved relatively quickly — but encourage the expectation that the path to solution will not always be obvious.

CLUES FOR TEACHING

Use questions to focus the child's attention on pertinent information given in the problem.

Encourage children to consider different strategies that might be used to solve the problem.

After they have reached a solution, encourage children to look back -- to reconsider their own thinking, to describe it, to note how they might have solved it differently -- or better, more efficiently (LeBlanc, 1977).

CLUE FOR TEACHING

Because these factors make problems difficult does not mean that we should avoid such problems. On the contrary, children need experiences with a range of problems which contain elements which make them difficult. That, after all, is problem solving! We should, however, be wary about giving children problems which are at too high a difficulty level for their capabilities: we do want them to be able to solve most (though not necessarily all) of the problems they meet.

CLUE FOR TEACHING

Children can profit from some instruction directed toward key words -- but this instruction should point out the dangers of relying on key words, as well as on their usefulness.

CLUE FOR TEACHING

Provide practice on problems with extraneous data. Most real-life problems are like this. Try searching newspapers for items: then pose a question which will demand that extraneous information be disregarded.

CLUE FOR TEACHING

Have children use — or make available for them to use -- manipulative materials, pictures, diagrams, etc., etc., etc.

CLUE FOR TEACHING

Flexibility in problem solving is a type of learned behavior. Students exposed to a variety of problems are able to make a smoother transition to new problems than those who are given practice only on many similar problems. Questioning may also contribute to the development of flexibility in children's problem solving behaviors (Cunningham, 1966).

CLUE FOR TEACHING

As one related example of this point, Smith (1973) suggests that problems involving related theorems should be alternated so that students are made aware of the essential elements of a theorem and do not develop a "set" to use a particular theorem when it is not applicable.

CLUES FOR TEACHING

Several general clues were provided by Flener (1978, p. 13):

- Whenever possible, embed a teaching/learning experience in a problem solving format.
- Think in terms of hints or suggestions rather than absolute procedures to be followed.
- Experiment by giving students less help than usual.
- Do not be misled by the immediate benefits of structural [expository] teaching -- the long-range benefits of teaching through problem solving may be higher.

CLUES FOR TEACHERS

It appears that techniques which attempt to "program" the solver to follow a fixed sequence of steps are not very effective. . . Experience and some research suggest, however, that certain heuristic procedures which will improve mathematical problem-solving performance can be learned -- provided the teacher illustrates how the procedures work; gives ample opportunity for discussion, practice, and reflection; and supports and encourages the learner's efforts. (Kilpatrick, 1978, p. 191)

CLUES FOR TEACHING

A quarter of a century ago, Brownell (1942) developed a list of suggestions for teachers. Three of them seem particularly appropriate in this section on concluding clues:

- To be most fruitful, practice in problem solving should not consist in repeated experiences in solving the same problems with the same techniques, but should consist of the solution of different problems by the same techniques and the application of different techniques to the same problems.
- A problem is not necessarily 'solved' because the correct response has been made. A problem is not truly solved unless the learner understands what he [or she] has done and knows why his [or her] actions were appropriate.
- Instead of being "protected" from error, the child should many times be exposed to error and be encouraged to detect and to demonstrate what is wrong, and why.

Lankford, Francis G., Jr. Some Computational Strategies of Seventh Grade Pupils. Final Report, USOE Grant Number OEG-3-72-0035, October 1972. ERIC: ED 069 496.

Some characteristics of poor computers (pp. 38-40)

1. Poor computers' stock of whole number facts is limited. They rely heavily on a few retained facts such as doubles, or products with 5 as one factor, from which to derive unknown combinations. They make extensive use of counting to make combinations.

2. Poor computers often make errors in whole number oprations when their counting, or other derivations of unknown combinations, become too involved for their short memory spans.

3. Poor computers have much more trouble with fractions than with whole numbers. Primitive methods such as counting are not as useful in fractions as in whole numbers . . .

4. Poor computers have difficulty remembering the conventional operational algorithms--especially in fractions. Moreover, they have difficulty in matching those they do remember with the right exercise. So they devise simple, and what seems to them as obvious, procedures such as adding numerators and then adding denominators for the sum of two simple fractions.

5. When poor computers encounter difficulty with an improvised algorithm, they often switch to something else that will produce an answer, however remote from the proper procedure it may be.

6. Poor computers tend to rely more on aids to memory.

7. What appear to be careless errors of poor computers are often supported by a reason--even if faulty.

8. Poor computers have great difficulty with long division . . . They will make several trial multiplications aside in an effort to find a quotient digit. The factors they choose to multiply the divisor by are otent arbitrarily chosen. In the exercise $84/\overline{93}$ a pupil multiplied aside 48 by 2, by 3, by 5, by 7, by 8, and by 9 in an attempt to find the correct quotient digit. In long division, other poor computers will design unorthodox algorithms that yield incorrect answers. An example is the practice of dividing by one digit of the divisor at a time.

9. Poor computers often did not hesitate to reverse minuend and subtrahend in subtraction.

10. Poor computers frequently confuse 0, and 1, as in 15 3/4 + 3/4 = 15 0/4 or in rewriting 708 as 7-9-18.

11. Poor computers are quite likely to be confused in arranging the partial products of multiplication--especially when the factors contain zeros.

Recommendations (Lankford, 1972, pp. 42-43)

1. Recorded interviews with pupils as they compute should be more widely used in research studies. They should also be used by the practicing teacher to the extent that time permits. It is suggested that early each fall a teacher do three to five interviews in each class taught. Pupils should be selected for interviews on the basis of their written work. For the interview, a variety of simple exercises should be prepared covering computational skills which have been studied previously. . . . It is expected that these sample interviews will give a teacher much clearer insights into the background of his pupils than would otherwise be likely. Moreover, these insights are likely to be helpful in planning further work with the pupils interviewed as well as the others in the class.

2. Supervisors should also learn the technique of interviewing and use it to prepare taped recordings for use in in-service meetings with teachers.

3. Some school systems have experimented with the employment of what have been called "diagnostic-prescriptive teachers." These full-time special teachers work individually with pupils referred to them by the regular classroom teacher. When the pupil goes back to his regular class, the "diagnostic-prescriptive teacher" sends to his teacher a diagnosis of his learning difficulties and some suggestions for correcting them. . . .

4. Teachers should encourage pupils to reveal individual strategies in their day-to-day computational exercises. They should not conceal their individual strategies through fear of ridicule. Rather there should be recognition for originality in thinking.

5. Teachers should give much more attention to teaching pupils to check the reasonableness of answers.

⋮

7. Many pupils need a lot of help in learning to compute and much of this help must be individualized. . . . New approaches to the teaching of computational skills should be seriously studied.

8. The individual interview technique should be used to ascertain the nature of arithmetical concepts held by pupils as well as the computational strategies they use.

⋮

ng answers--Fractions (Lankford, 1972, p. B-1)*

3/4 + 5/2 = ___

Answers	
8/4 = 2	"5 + 3 = 8. You don't add the bottom numbers because 2 will go into 4. (23 pupils)
8/6	3 and 5 is 8 ; 4 and 2 is 6. (May or may not be reduced.) (36 pupils)
8/8	5 + 3 = 8 ; 2 plus 4 is 8. (3 pupils)
68	3 and 5 is 8 ; 4 and 2 is 6.
15/8	5 and 3 is 15 ; 4 and 2 is 8. (2 pupils)
7/7	3 + 4 = 7 ; 2 + 5 = 7. (2 pupils)
7/6	3 + 5 is 7 ; 4 and 2 is 6.
3 1/8	Chose 8 as C.D. Then 4 x 3 = 12, so 3/4 = 12/8 ; 5 x 2 is 10, so 5/2 = 10/8 ; 12/8 + 10/8 = 22/8 = 3 1/8 (3 x 8 = 21, with 1 left over).
86	5 + 3 = 8 ; 4 + 2 = 6. (2 pupils)
2/8	Chose 8 as C.D. "8 goes into 2 four times and 4 goes into 5 one time." So 5/2 became 1/8. "8 goes into 4, two times, 2 will go into 3 one time," so 3/4 became 1/8. Then 1/8 + 1/8 = 2/8.
16/8 = 2	Chose 8 as C.D. Then 3/4 = 6/8 and 5/2 = 10/8, and 6/8 + 10/8 = 16/8.
14	"5 over 2 is 7 ; 3 over 4 is 7 ; 7 + 7 = 14."
2	3/4 = 3/4 and 5/2 = 5/4. Then 3 + 5 = 8 over 4 for 8/4 = 2.
2 1/4	Got 13/4 correctly. Decided, by counting, there are two 4's in 13 with one left over.
3/2	5/2 = 5/4 ; 3/4 = 3/4. Then 5 + 3 = 8 over 4 and 8/4 = 3/2.
3	3/4 = 3/4 ; 5/2 = 2 1/2 ; chose 8 as C.D. For 3/4 said "4 goes into 8 two times and 3 goes into 8 two times." So wrote 2/2 for 3/4. Then "2 goes into 8 four times, and 1 goes into 8 eight times" so wrote 8/4 for 2 1/2. Then divided $4/\overset{2}{\overline{8}}$. Added this 2 to 1 from 2/2 of 3/4 for answer of 3.
8 3/2	Wrote 5/2 as 20/8 and 3/4 as 6/8, added for 26/8. 26 ÷ 8 = 8 R2 wrote as 8 3/2.
24/4	Chose 4 as C.D. Then for 3/4 said "4 times 1 equals 4, and 1 plus 3 is 4" so 4/4. For 5/2 said "2 x 2 = 4 and 4 x 5 = 20" so 20/4. Then 4/4 + 20/4 = 24/4.
4 1/4	Got sum of 13/4 correctly. Then "4 into 13 goes 4 with 1 left over."
59	"4 and 5 is 9 ; 3 and 2 is 5. Answer would be 95" but wrote as 59.

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[*] Appendices contain many additional pages citing wrong answers.

TABLE 9.6
FRACTIONS

Item	Source	Grade (Age)[a]											
		1	2	3	4	5	6	7	8(13)	9	10	11(17)	12
$\frac{1}{4} + \frac{2}{4}$	NLSMA				17			89					
$\frac{2}{3} + \frac{4}{3}$[b]	MI							65(66)[c,d]					
$2\frac{1}{2} + 1\frac{1}{2}$	CT						51	78	85	91	89	92	96
$4\frac{1}{3} + 2\frac{1}{3}$[b]	MI							61(56)[c,d]					
$\frac{7}{9} - \frac{5}{9}$	CT	0	1	4	21	52							
$\frac{14}{16} - \frac{5}{16}$	NLSMA							90	92			86	
$3\frac{5}{6} - \frac{1}{6}$[b]	MI							63(67)[c,d]					
$\frac{3}{8}$ $+\frac{1}{2}$	NLSMA				62				81				
$6\frac{1}{4}$ $1\frac{5}{8}$ $+4\frac{1}{2}$	CT						33	69	77	77	77	83	81
$\frac{11}{12} - \frac{5}{6}$	NLSMA							82	87			84	
$\frac{7}{8} - \frac{1}{2}$	MI							33					
$\frac{9}{10} - \frac{101}{1000}$	NLSMA							45	55			56	
$\frac{1}{2} + \frac{1}{3}$	NAEP								42			66	
$\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$	MI							46					
$10\frac{1}{4}$ $-7\frac{2}{3}$	CT						18	53	67	53	53	51	54
$\frac{1}{2} \times \frac{1}{4}$	NAEP								62			74	
$\frac{3}{4} \times \frac{8}{9}$	NLSMA						67		74				
$5 \times \frac{2}{25}$	NLSMA						65		76				
$\frac{1}{8}$ of 30	CT						62	79	90	91	89	95	98
$\frac{7}{9} \times 4$	NLSMA						61		70				
$6 \times 3\frac{7}{8}$	CT						2	21	36	34	33	32	31

[a, b, c] See notes on table 9.2.

[d] The data are from 1973 (1974).

Bright, George W. Assessing the Development of Computation Skills. In Developing Computational Skills (edited by Marilyn N. Suydam). In 1980 NCTM Yearbook. Reston, Virginia: National Council of Teachers of Mathematics, 1978.

TABLE 9.5
DIVISION

Item	Source	Grade (Age)[a]									
		4(9)	5	6	7	8(13)	9	10	11(17)	12	Adult
$3\overline{)69}$	NLSMA	80			94						
$7\overline{)91}$	NLSMA	61			88						
$7\overline{)910}$	MI				71(76)[c,d]						
$5\overline{)125}$	NAEP	15				89			93		93
$6\overline{)846}$	NLSMA	53			84						
$6\overline{)7356}$	NLSMA	32			78						
$9\overline{)4527}$	CT			42	72	83	83	82	81	92	
5 digits ÷ 1 digit, no remainder	NAEP	5				67			78		77
3 digits ÷ 2 digits, no remainder	NAEP					68			85		
$32\overline{)9792}$	NLSMA		44		67	69					
$24\overline{)482}$	NLSMA		61		66	72					
$500\overline{)3029}$	NLSMA		60		84	90					

[a,b,c] See notes on table 9.2

[d] The data are from 1973 (1974).

(p. 156)

NATIONAL COUNCIL OF SUPERVISORS OF MATHEMATICS POSITION PAPER ON BASIC MATHEMATICAL SKILLS

INTRODUCTION

The currently popular slogan "Back to the Basics" has become a rallying cry of many who perceive a need for certain changes in education. The result is a trend that has gained considerable momentum and has initiated demands for programs and evaluations which emphasize narrowly defined skills.

Mathematics educators find themselves under considerable pressure from boards of education, legislatures, and citizens' groups who are demanding instructional programs which will guarantee acquisition of computational skills. Leaders in mathematics education have expressed a need for clarifying what are the basic skills needed by students who hope to participate successfully in adult society.

The narrow definition of basic skills which equates mathematical competence with computational ability has evolved as a result of several forces:

1. Declining scores on standardized achievement tests and college entrance examinations;
2. Reactions to the results of the National Assessment of Educational Progress;
3. Rising costs of education and increasing demands for accountability;
4. Shifting emphasis in mathematics education from curriculum content to instructional methods and alternatives;
5. Increased awareness of the need to provide remedial and compensatory programs;
6. The widespread publicity given to each of the above by the media.

This widespread publicity, in particular, has generated a call for action from governmental agencies, educational organizations, and community groups. In responding to these calls, the National Institute of Education adopted the area of basic skills as a major priority. This resulted in a Conference on Basic Mathematical Skills and Learning, held in Euclid, Ohio, in October, 1975.

The National Council of Supervisors of Mathematics (NCSM), during the 1976 Annual Meeting in Atlanta, Georgia, met in a special session to discuss the Euclid Conference Report. More than 100 members participating in that session expressed the need for a unified position on basic mathematical skills which would enable them to provide more effective leadership within their respective school systems, to give adequate rationale and direction in their tasks of implementing basic mathematics programs, and to appropriately expand the definition of basic skills. Hence, by an overwhelming majority, they mandated the NCSM to establish a task force to formulate a position on basic mathematical skills. This statement is the result of that effort.

RATIONALE FOR THE EXPANDED DEFINITION

There are many reasons why basic skills must include more than computation. The present technological society requires daily use of such skills as estimating, problem solving, interpreting data, organizing data, measuring, predicting, and applying mathematics to everyday situations. The changing needs of society, the explosion of the amount of quantitative data, and the availability of computers and calculators demand a redefining of the priorities for basic mathematics skills. In recognition of the inadequacy of computation alone, NCSM is going on record as providing both a general list of basic mathematical skills and a clarification of the need for such an expanded definition of basic skills.

Any list of basic skills must include computation. However, the role of computational skills in mathematics must be seen in the light of the contributions they make to one's ability to use mathematics in everyday living. In isolation, computational skills contribute little to one's ability to participate in mainstream society. Combined effectively with the other skill areas, they provide the learner with the basic mathematical ability needed by adults.

DEFINING BASIC SKILLS

The NCSM views basic mathematical skills as falling under ten vital areas. The ten skill areas are interrelated and many overlap with each other and with other disciplines. All are basic to pupils' development of the ability to reason effectively in varied situations.

This expanded list is presented with the conviction that mathematics education must not emphasize computational skills to the neglect of other critical areas of mathematics. The ten components of basic mathematical skills are listed below, but the order of their listing should not be interpreted as indicating either a priority of importance or a sequence for teaching and learning.

Furthermore, as society changes our ideas about which skills are basic also change. For example, today our students should learn to measure in both the customary and metric systems, but in the future the significance of the customary system will be mostly historical. There will also be increasing emphasis on when and how to use hand-held calculators and other electronic devices in mathematics.

TEN BASIC SKILL AREAS

Problem Solving

Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with non-textbook problems. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. In solving problems, students need to be able to apply the rules of logic necessary to arrive at valid conclusions. They must be able to determine which facts are relevant. They should be unfearful of arriving at tentative conclusions and they must be willing to subject these conclusions to scrutiny.

Applying Mathematics to Everyday Situations.

The use of mathematics is interrelated with all computation activities. Students should be encouraged to take everyday situations, translate them into mathematical expressions, solve the mathematics, and interpret the results in light of the initial situation.

Alertness to the Reasonableness of Results

Due to arithmetic errors or other mistakes, results of mathematical work are sometimes wrong. Students should learn to inspect all results and to check for reasonableness in terms of the original problem. With the increase in the use of calculating devices in society, this skill is essential.

Estimation and Approximation

Students should be able to carry out rapid approximate calculations by first rounding off numbers. They should acquire some simple techniques for estimating quantity, length, distance, weight, etc. It is also necessary to decide when a particular result is precise enough for the purpose at hand.

Appropriate Computational Skills

Students should gain facility with addition, subtraction, multiplication, and division with whole numbers and decimals. Today it must be recognized that long, complicated computations will usually be done with a calculator. Knowledge of single-digit number facts is essential and mental arithmetic is a valuable skill. Moreover, there are everyday situations which demand recognition of, and simple computation with, common fractions.

Because consumers continually deal with many situations that involve percentage, the ability to recognize and use percents should be developed and maintained.

Geometry

Students should learn the geometric concepts they will need to function effectively in the 3-dimensional world. They should have knowledge of concepts such as point, line, plane, parallel, and perpendicular. They should know basic properties of simple geometric figures, particularly those properties which relate to measurement and problem-solving skills. They also must be able to recognize similarities and differences among objects.

Measurement

As a minimum skill, students should be able to measure distance, weight, time, capacity, and temperature. Measurement of angles and calculations of simple areas and volumes are also essential. Students should be able to perform measurement in both metric and customary systems using the appropriate tools.

Reading, Interpreting, and Constructing Tables, Charts, and Graphs

Students should know how to read and draw conclusions from simple tables, maps, charts, and graphs. They should be able to condense numerical information into more manageable or meaningful terms by setting up simple tables, charts, and graphs.

Using Mathematics to Predict

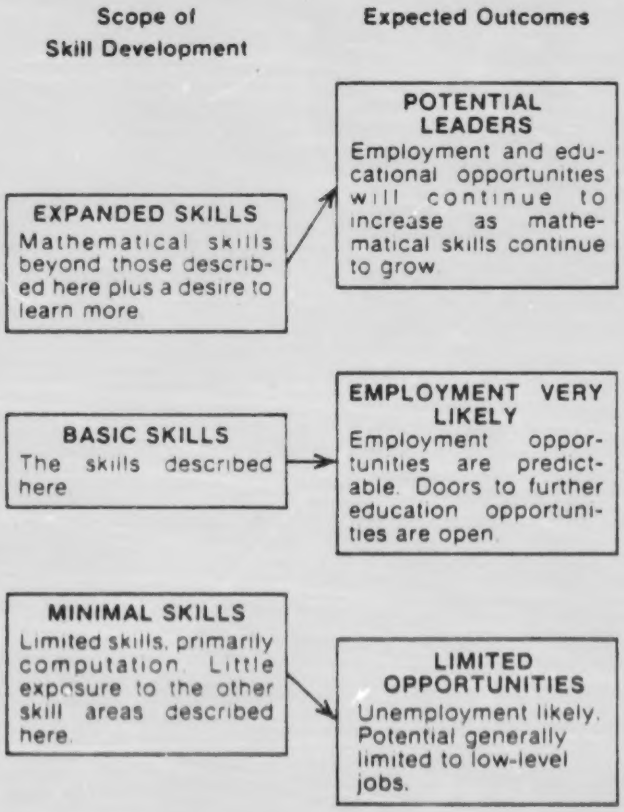
Students should learn how elementary notions of probability are used to determine the likelihood of future events. They should learn to identify situations where immediate past experience does not affect the likelihood of future events. They should become familiar with how mathematics is used to help make predictions such as election forecasts.

Computer Literacy

It is important for all citizens to understand what computers can and cannot do. Students should be aware of the many uses of computers in society, such as their use in teaching/learning, financial transactions, and information storage and retrieval. The "mystique" surrounding computers is disturbing and can put persons with no understanding of computers at a disadvantage. The increasing use of computers by government, industry, and business demands an awareness of computer uses and limitations.

BASIC SKILLS AND THE STUDENT'S FUTURE

Anyone adopting a definition of basic skills should consider the "door-opening/door-closing" implications of the list. The following diagram illustrates expected outcomes associated with various amounts of skill development.



Scope of Skill Development

Expected Outcomes

EXPANDED SKILLS
Mathematical skills beyond those described here plus a desire to learn more

POTENTIAL LEADERS
Employment and educational opportunities will continue to increase as mathematical skills continue to grow

BASIC SKILLS
The skills described here

EMPLOYMENT VERY LIKELY
Employment opportunities are predictable. Doors to further education opportunities are open.

MINIMAL SKILLS
Limited skills, primarily computation. Little exposure to the other skill areas described here.

LIMITED OPPORTUNITIES
Unemployment likely. Potential generally limited to low-level jobs.

MINIMUM ESSENTIALS FOR HIGH-SCHOOL GRADUATION

Today some school boards and state legislatures are starting to mandate mastery of minimum essential skills in reading and mathematics as a requirement for high-school graduation. In the process, they should consider the potential pitfalls of doing this without an appropriate definition of "basic skills." If the mathematics requirements are set inordinately high, then a significant number of students may not be able to graduate. On the other hand, if the mathematics requirements are set too low and mathematical skills are too narrowly defined, the result could be a sterile mathematics program concentrating exclusively on learning of low-level mathematical skills. This position paper neither recommends nor condemns minimal competencies for high-school graduation. However, the ten components of basic skills stated here can serve as guidelines for state and local school systems that are considering the establishment of minimum essential graduation requirements.

DEVELOPING THE BASIC SKILLS

One individual difference among students is style or way of learning. In offering opportunities to learn the basic skills, options must be provided to meet these varying learning styles. The present "back-to-basics" movement may lead to an emphasis on drill and practice as a way to learn.

Certainly drill and practice is a viable option, but it is only one of many possible ways to bring about learning and to create interest and motivation in students. Learning centers, contracts, tutorial sessions, individual and small-group projects, games, simulations and community-based activities are some of the other options that can provide the opportunity to learn basic skills. Furthermore, to help students fully understand basic mathematical concepts, teachers should utilize the full range of activities and materials available, including objects the students can actually handle.

The learning of basic mathematical skills is a continuing process which extends through all of the years a student is in school. In particular, a tendency to emphasize computation while neglecting the other nine skill areas at the elementary level must be avoided.

EVALUATING AND REPORTING STUDENT PROGRESS

Any systematic attempt to develop basic skills must necessarily be concerned with evaluating and reporting pupil progress.

In evaluation, test results are used to judge the effectiveness of the instructional process and to make needed adjustments in the curriculum and instruction for the individual student. In general, both educators and the public have accepted and emphasized an overuse of and overconfidence in the results of standardized tests. Standardized tests yield comparisons between students and can provide a rank ordering of individuals, schools, or districts. However, standardized tests have several limitations including the following:

a. Items are not necessarily generated to measure a specific objective or instructional aim.
b. The tests measure only a sample of the content that makes up a program; certain outcomes are not measured at all.

Because they do not supply sufficient information about how much mathematics a student knows, standardized tests are not the best instruments available for reporting individual pupil growth. Other alternatives such as criterion tests or competency tests must be considered. In criterion tests, items are generated which measure the specific objectives of the program and which establish the student's level of mastery of these objectives. Competency tests are designed to determine if the individual has mastered the skills necessary for a certain purpose such as entry into the job market. There is also need for open-ended assessments such as observations, interviews, and manipulative tasks to assess skills which paper and pencil tests do not measure adequately.

Reports of pupil progress will surely be made. But, while standardized tests will probably continue to dominate the testing scene for several years, there is an urgent need to begin reporting pupil progress in other terms, such as criterion tests and competency measures. This will also demand an immediate and extensive program of inservice education to instruct the general public on the meaning and interpretation of such data and to enable teachers to use testing as a vital part of the instructional process.

Large scale testing, whether involving all students or a random sample, can result in interpretations which have great influence on curriculum revisions and development. Test results can indicate, for example, that a particular mathematical topic is being taught at the wrong time in the student's development and that it might better be introduced later or earlier in the curriculum. Or, the results might indicate that students are confused about some topic as a result of inappropriate teaching procedures. In any case, test results should be carefully examined by educators with special skills in the area of curriculum development.

CONCLUSION

The present paper represents a preliminary attempt by the National Council of Supervisors of Mathematics to clarify and communicate its position on basic mathematical skills. The NCSM position establishes a framework within which decisions on program planning and implementation can be made. It also sets forth the underlying rationale for identifying and developing basic skills and for evaluating pupils' acquisition of these competencies. The NCSM position underscores the fundamental belief of the National Council of Supervisors of Mathematics that any effective program of basic mathematical skills must be directed not "back" but *forward* to the essential needs of adults in the present and future.

You are encouraged to make and distribute copies of this paper.

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January, 1977

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[3] President, NCSM
[4] Resource Person, National Institute of Education

Miller

USE YOUR CALCULATOR TO FIND THE MISSING NUMBERS IN EACH OF THESE MULTIPLICATION PROBLEMS. EACH ARROW IS POINTING TO A PLACE WHERE A NUMBER IS MISSING.

673 x 45 ← 26920 ←	967 x 84 ← ← 81228	← x 84 1824 ← 38304
846 x 79 ← ← ←	432 x ← ← ← 12528	374 x ← 3366 29920 ←
476 x ← ← ← 99484	476 x ← ← 38080 8 ←	← x 46 4182 ← ←
← x 78 ← 27790 ←	← x 360 ← 203400 ←	← x 307 5243 ← ←

Miller

72

LOOK AT WHAT CAL Q. LATE DID TO ESTIMATE THE SUM OF THE NUMBERS IN THE TOP ROW OF THE TABLE.
CAL THINKS: 50 + 80 + 60 + 20 + 80 + 40 = 330.
TRY ESTIMATING THE SUMS FOR THE NUMBERS IN THE OTHER ROWS BY USING CAL'S METHOD. USE A CALCULATOR TO FIND THE ACTUAL SUM AFTER EACH ESTIMATE.

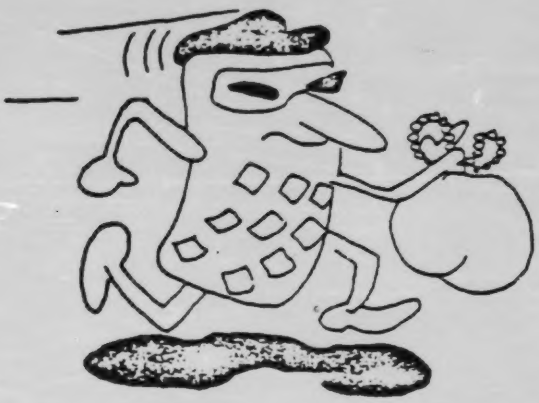
							ESTIMATE	ANSWER
48	79	61	19	82	43	→	330	332
97	69	44	13	36	71	→		
42	83	67	17	88	52	→		
87	99	57	52	61	69	→		
28	43	87	61	77	12	→		
29	39	49	59	69	84	→		
84	73	62	51	16	87	→		
23	79	63	14	56	37	→		
58	161	397	608	9	32	→		
196	302	731	576	87	42	→		
597	803	416	687	96	396	→		

CALCULATOR CAPERS

I

MRP

C-123



MAKE A 2 APPEAR IN THE DISPLAY.

MAKE ALL ZEROS APPEAR IN THE DISPLAY.

WHAT DOES THE "C" BUTTON DO?

ADD 12 AND 72 ON THE CALCULATOR.

WHAT IS THE LARGEST NUMBER YOU CAN SHOW ON THE CALCULATOR?

ADD 792 + 792 + 792. WHAT IS YOUR ANSWER?

DO 792 + 792 + 792 AS A MULTIPLICATION PROBLEM.

DO 192 + 192 + 192 + 192 + 192 AS A MULTIPLICATION PROBLEM.

FIND THE ANSWER TO 973 x 796.

WORK THE FOLLOWING PROBLEM TWO DIFFERENT WAYS ON THE CALCULATOR.

(274 + 184) x 492

FIND THE ANSWER TO THE SUBTRACTION PROBLEM 3975 - 2886.

HOW MANY TIMES CAN YOU SUBTRACT 26 FROM 168?

DIVIDE 45 BY 15.

SUBTRACT 15 FROM 45 AS MANY TIMES AS POSSIBLE. HOW MANY?

DID YOU GET THE SAME ANSWER FOR THE LAST TWO PROBLEMS?

DIVIDE 486 BY 54.

DIVIDE 9583 BY 63.

MAKE UP A DIVISION PROBLEM OF YOUR OWN AND WORK IT.

MAKE UP A MULTIPLICATION PROBLEM OF YOUR OWN AND WORK IT.

WORK THE FOLLOWING PROBLEM ON THE CALCULATOR. (265 x 75) + 3.

MAKE UP SOME PROBLEMS AND WORK THEM.

This activity is from the Oregon
Resource Project materials, now available
from Creative Publications.

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May 1979

The Use of Calculators in Pre-College Education: A State-of-the-Art Review*

Marilyn N. Suydam

While no dramatic changes have characterized the situation during the past year, the use of calculators in schools continues to increase. Requests for information on calculator materials, attendance at workshops, and correspondence from teachers using calculators indicate that calculators are "slowly but surely" being assimilated into the instructional process. Whether or not to use calculators in classrooms is still an issue for some teachers and parents, but people are more accustomed to having the aids available. The calculator's role as an instructional tool is being recognized: it saves time spent on tedious calculation, and allows more time for developing mathematical ideas and for considering interesting content and problems. A new position statement from the National Council of Teachers of Mathematics cites other ways in which the calculator can aid instruction; the Council "encourages the use of calculators in the classroom as instructional aids and computational tools". While many still believe that students must master computational facts and procedures for addition and other operations before they use calculators, others are finding that there is some purpose for using calculators at each level.

Extent of Use in Schools

One of the most frequently asked questions is, "How many students are using calculators in schools?" Few data to supply an answer to this question have been published. Several studies were cited in the Calculator Information Center 1978 state-of-the-art review; the results of one national survey provide additional information. This survey, conducted during the winter of 1977 by Iris Weiss, queried a random sample of 1177 schools representative of the total school population across the country. They indicated increasing use as grade level increased:

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* This report concerns the period since April 1978, when the Calculator Information Center published a previous state-of-the-art review.

	Grade Level			
	K-3	4-6	7-9	10-12
Schools having calculators	28%	36%	49%	77%
Teachers using calculators in science classes	2%	12%	10%	36%
Teachers using calculators in mathematics classes	6%	14%	30%	48%

Weiss added:

> Many teachers feel that calculators are not needed in their science classes (ranging from 47 percent in grades 10-12 to 69 percent in grades K-3), while between 14 and 19 percent indicate they are needed but not available. (p. 130)

For mathematics classes, the comparable data are:

	K-3	4-6	7-9	10-12
Calculators not needed	77%	44%	42%	33%
needed but not available	15%	39%	28%	18%

Unfortunately, these data are now two years old. It will be interesting to see what changes in the percentages occur as data are collected in 1979.

From a 1978 investigation with students in grades 2 through 6 in five midwestern states, Richard Shumway noted that the availability of calculators in the home increased during the project. At the beginning of the study, about 80% of the parents and 25% to 30% of the children had calculators at home. At the end, 90% of the parents and 45% of the children had calculators at home. This parallels previous information that when teachers encourage use of calculators in instruction, children are likely to acquire them to use both in and out of school. As George Immerzeel, who has conducted workshops for many teachers, noted: "Almost any teacher who asks children to bring calculators finds that they do."

In several school districts, competency with calculators is included on the list of minimal competencies expected for graduation. The Ramona Unified School District in San Diego County, California, for instance, specified "The student can use a calculator to perform basic operations (+, -, x, ÷)" as one of nine competencies necessary to meet the goal of demonstrating "the ability to use basic mathematical processes necessary to function effectively in society."

Research on Calculator Effects

Almost 100 studies on the effects of calculator use have been conducted during the past four or five years. This is more investigations than on almost any other topic or tool or technique for mathematics instruction during this century, and calls attention to the intensive interest about this potentially valuable tool. Many of these studies had one goal: to ascertain whether or not the use of calculators would harm students' mathematical achievement. The answer continues to be "No". The calculator does not appear to affect achievement adversely. In all but a few instances, achievement scores are *as high or higher* when calculators are used for mathematics instruction (but not on tests) than when they are not used for instruction. Thus, many researchers working with students at all levels have paralleled the conclusion of Shumway and his co-investigators that "There were no measurable detrimental effects associated with use of calculators for teaching mathematics."

While a number of studies have replicated the finding that use of calculators does not appear to have harmful effects on the mathematical achievement of students who use calculators, many of the studies have not carefully documented how the students or teachers use the calculators. Often the calculator is used "as the teacher or student sees fit" -- which may mean only for checking answers attained by paper-and-pencil computation or, worse, for activities which in actuality merely affirm that the calculator can calculate. Other researchers have turned to a more careful analysis of how the calculator is used.

Some of these researchers are exploring the role of the calculator in relation to problem solving, a primary goal of mathematics instruction. Billy Hopkins found that calculators helped students in a ninth-grade basic mathematics course to achieve better problem-solving scores than non-calculator users did. A study by Michael Kasnic provides additional evidence that calculators helped such students "compete successfully" with students of higher ability.

Charlotte Wheatley considered the effect of calculator use on problem-solving strategies employed by children in grade 5, while Glendon Blume reported on how seventh graders solved equations. It appears that different strategies and solution methods are used with calculators than are used without calculators. In particular, the calculator makes the exploration of hypotheses feasible. In conjunction with another problem-solving study, Marguerite Mason reported that "There was no evidence that the students become calculator dependent", thus providing a response to a concern of some parents.

Development of Instructional Materials

Teachers who want to use calculators find that they need specific materials if they are to integrate calculators into instruction. Therefore, individuals and groups have continued the development of materials designed specifically to make use of the unique capabilities of the calculator to promote desired mathematical objectives. One such instance was funded by the National Institute of Education. Gerald Rising and others at the State University of New York at Buffalo are developing instructional materials for students in grades 11 and 12 in which the use of calculators is integrated. Problem solving and the development of mathematical skills and ideas are furthered by using the calculator.

In a number of school districts around the country, the development of materials is also proceeding. Funding from the Elementary and Secondary Education Act Title IV-C has aided districts from New England to California. Modules to accompany the regular curriculum, activity cards which supplement non-calculator work, and other materials are being designed by these groups. Most have in-service education components, in which teachers in the district are trained to use the materials.

For some years, George Immerzeel has been exploring the problem-solving strategies which children use and designing instructional materials to help them become better problem solvers. With his colleagues at the University of Northern Iowa and funding from Title IV-C, he has developed workbooks and sets of problems which incorporate the use of calculators to teach such specific problem-solving strategies as estimation and the use of tables and graphs.

Other groups have focused on meeting the needs of teachers for experiences with calculators and calculator materials. The North Carolina Department of Public Instruction has developed a kit of materials for in-service education. Transparencies, sets of problems, and other materials are included to aid in conducting workshops for teachers. At meetings of such organizations as the National Council of Teachers of Mathematics and the School Science and Mathematics Association, workshops are included on the program. Of even more interest are the numerous school districts across the country which have initiated workshops for teachers (and, in some cases, parents).

An increasing number of articles present ideas for integrating the calculator into the on-going instructional program. For instance, an article by Manfried

Olson provides examples that can be used to stimulate conjectures, lending themselves to algebraic proof. A calculator enables students to test cases which would have been fruitless, if not impossible, without a calculator.

The needs of specific groups of students are also being considered. A group of educational agencies, for example, has worked together to develop materials to help visually handicapped persons use calculators. Thus far they have produced some materials for adolescents and young adults, providing a self-instructional program with a focus on daily living and real-life problems. Similar materials are sought by those with other handicaps. The needs of low achievers at both the pre-college and college levels have also been considered; materials and procedures for calculator use by students who have not been particularly successful in mathematics are being developed.

Continuing Concerns

In June 1976, a conference was held to explore needed research and development with calculators; in January 1979, a second conference was sponsored by the National Institute of Education to specify in more detail the points made at the first conference as well as to explore additional needs. The participants strongly suggest that a reiteration of the concerns expressed at the first conference is vital. While a report of the conference has not yet been developed, it appears that, among other points, it will stress the need for:

- Research on how calculators can develop mathematical skills and concepts at all levels.
- Development of instructional materials which integrate the use of calculators
- Continuing evaluation of the effects of calculators on achievement and attitudes

- Continuing concern for the needs of all types of learners as they use calculators
- Emphasis on providing preservice and in-service teachers with calculator experiences

Another continuing concern is with the role of calculators on tests. In several other countries, calculator use has been allowed on standardized tests; this is not yet true in the U.S. There is a stalemate at present: it is not appropriate to use calculators on normed tests, since both tests and norms were developed without calculators being used. On the other hand, tests which allow the use of calculators will not be available until calculators are in much wider use. Teachers, however, have become aware of the need to develop classroom tests which assess mathematical ideas, rather than computational accuracy, when students use calculators.

Concluding Comment

Slowly but surely, the calculator is being incorporated into the school program at all levels. It is being recognized as an instructional tool which has certain capabilities. But it is not a pancea: it cannot resolve all the difficulties in mathematics instruction. Moreover, it has certain limitations; teachers must accept the responsibility for teaching children how and when to use calculators, and thus to be aware of its limitations. After all, students now in school will have calculators, or a similar computational tool, to use for the rest of their lives.

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Iowa Problem-Solving Project (ESEA Title IV-C). Joan Duea, George Immerzeel, Earl Ockenga, John Tarr, and Jack Wilkinson, Price Laboratory School, University of Northern Iowa, Cedar Falls, 1978.

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Olson, Melfried. Using Calculators to Stimulate Conjectures and Algebraic Proofs. Mathematics Teacher 72: 288-289; April 1979.

Report of the Conference on Needed Research and Development on Hand-Held Calculators in School Mathematics. National Institute of Education and National Science Foundation, 1977.

Rising, Gerald R.; Krist, Betty J.; Roesch, Carl; and Jewell, Wallace. Using Calculators in Mathematics. National Institute of Education Contract No. 400-78-0013. State University of New York at Buffalo, 1978.

Shumway, Richard J.; Wheatley, Grayson H.; White, Arthur L.; Coburn, Terrence G.; Reys, Robert E.; and Schoen, Harold L. Initial Impact of Calculators in Elementary School Mathematics. Draft of manuscript submitted for publication, 1979.

Weiss, Iris R. Report of the 1977 National Survey of Science, Mathematics, and Social Studies Education. Final Report, National Science Foundation Contract No. C7619848. Research Triangle Park, North Carolina: Center for Educational Research and Evaluation, March 1978.

The reports by Glendon Blume, Marguerite Mason, and Charlotte Wheatley are found in Research Reporting Sessions, 57th NCTM Annual Meeting. Columbus: ERIC/SMEAC, April 1979.

Copies of the NCTM Position Statement on Calculators in the Classroom are available from the National Council of Teachers of Mathematics, 1906 Association Drive, Reston, Virginia 22091.

Practical Ways to Teach the Basic Mathematical Skills. Richmond:
Virginia Council of Teachers of Mathematics, 1978. [2332 Scarsborough
Dirve, Richmond, 23235]

Problem solving (pp. 14-15):

Strategy: ____________________

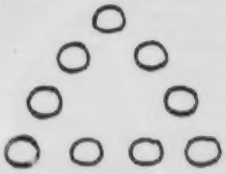
How many ways can 15 pennies be placed in 4 separate piles so that no 2 piles have the same amount?

How many ways can you use standard coins to make 87¢?

What numbers between 1 and 100 are not divisible by 2, 3, 5, or 7? Is there a way you can easily extend your method to find such numbers between 100 and 200?

Strategy: ____________________

Place the digits from 1 to 9 to the triangle so that each side has as sum of 20.



In the alphametics below, each letter represents a specific digit from 0 to 9.

$$\begin{array}{r} \text{ONE} \\ \text{TWO} \\ \underline{+\text{FIVE}} \\ \text{EIGHT} \end{array} \qquad\qquad \begin{array}{r} \text{FIVE} \\ \underline{-\text{FOUR}} \\ \text{ONE} \end{array}$$

A certain bag of marbles can be broken up into 2, 3, 4, 5, or 6 separate but equal size piles with one marble left over each time. It can also be separated into 7 piles evenly with none left. How many marbles are in the bag?

Strategy: ____________________

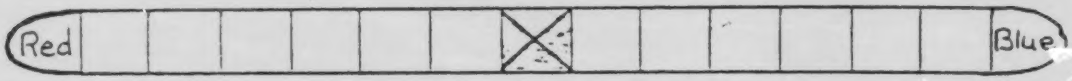
How many squares are on a standard chess board? (The answer is not 64.)

Each day 7 men sit at a round table for lunch. Each day they try to sit in a different order around the table. How many such orders are there is any rotation of all 7 men is not counted as different?

(pp. 22-23)

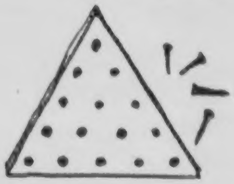
a. Nim. In this strategy game, piles of counters are set out and each player in turn removes all or some of the counters from any one pile. The object is to force the opponent to take the last counter.

b. Board games. A number of simple board games for one or two players are easily made and provide lots of opportunity to devise strategies.



"Slide and Jump" starts with six counters on each side of the "X." The object is to reverse the positions. Blue markers can move left and Red markers right. Moves can be a slide of one space to a vacant square or a jump over one marker (not removed) to a vacant square. The real challenge is to work toward a minimum number of moves and then extend the result to more counters.

The "Golf Tee Triangle" game starts with 14 tees in any of the 15 holes in the triangle. Tees are removed by jumping them with other tees. The object is to leave only one tee.



Other excellent board games which have stood the test of time are Nine Men Morris, Go (a Chinese game), Chinese Checkers, and Hex. Othello is a newer commercial game in the same spirit. (See articles by Gessel, Ruderman, and Timmons for additional easily made games of this type.) Checkers, Chess, and Backgammon should also be included in a good collection of think games.

c. Mastermind. This award wining game is easily learned. One player hides a code of colored pegs and through a series of guesses and coded responses the other player discovers the code. Few games of such simplicity employ quite as much deductive logical reasoning. A pencil and paper game called Pico-Fumi (see Gessel, et al.) is equally good. In fact, Pico-Fumi can be made even more difficult than Mastermind and requires no materials.

(pp. 29-32)

Garland has a cold. The doctor gave him a prescription to take to the drugstore. The directions on the medicine container he received reads: "Take one capsule every four hours for three days." He takes the first capsule Monday at 1 o'clock in the afternoon and follows the prescribed schedule. When will he take the last capsule?

The Browns are expecting friends for dinner at 5:30 p.m. Sally's menu includes a meat dish which requires 1 hour and 15 minutes to cook; a vegetable which requires 20 minutes to cook; and a pie which bakes for 45 minutes. At what time should Sally begin cooking each item if she wants it to be ready exactly on time? Make a schedule for Sally to follow.

Mr. Elton bought a battery for his car. The warranty stated that it should be useful for three years and if it fails within this period the buyer can receive a portion of the amount paid for the battery toward the cost of a new battery (purchased from their company). The amount that would be credited toward the cost of a new battery is based on the amount of time that remains in the warranty period. Mr. Elton bought a battery for $54.00. After using it for 26 months the battery failed. How much credit will he get toward the purchase of a new battery?

An advertisement at a tire store read: "Two tires for $59.00 with $5.00 off for each trade-in." Mr. Crane bought four tires but traded only two tires. What amount did he pay?

The Market Basket. A list of items included in a typical market basket is given to each student. Each student visits a supermarket and records for each listed item the brand, the price, and the quantity. The total cost of the basket of groceries is computed by each student.

Sally wants to buy a pair of jeans that are priced at $15.95. How much tax must she pay if the tax rate is 5%? What is the total cost of the jeans?

Ajax supermarket sells three oranges for 25 cents. The Big Horn supermarket sells a dozen oranges for 96 cents. At which store would you buy a dozen oranges? Why?

Select from newspapers or sales catalogs pictures of several items that include their prices. Arrange the items on poster board. Assign a number to each item. Students are asked to observe the display and find the amount of sales tax and total cost of each item.

A variation of activity 5 is one in which transparencies are used for displaying items. As the items and their prices are shown, the teacher receives from students quick estimates of the amount of sales tax required for each item.

This is a problem for an activity center, based on one presented by Bruni and Silverman in the Arithmetic Teacher, (January, 1977).

Materials and equipment needed:

a. a list of zipcodes,
b. a zone chart for parcel post priority mail,
c. a small mail scale and several packages which have been addressed for mailing.

Objective: Determine the postage for each package. See task card and record card below.

Task Card

1. Select a package.
2. Weight the package on the postal scale.
3. Determine the official zone.
4. Use the parcel post table to determine the correct cost.
5. Complete the record card.

Record Card

Student's Name

Package	Weight	Zone	Cost
1.			
2.			
3.			
4.			
5.			

Using a table of rates for electricity in your local community, have students compute the cost of consuming various numbers of kilowatts of electricity.

Make a display of pictures showing various electrical items that are used in homes. Show for each item a number of kilowatts of electricity normally used in operating the item for a month. Students calculate the monthly cost of operating each item using a rate schedule provided by the local company that supplies electricity.

Provide students with a rate schedule for the cost of electricity and a listing of typical household items which includes the amount of electricity consumed in operating the items for 12 hours. Students determine which items they can use and the number of hours they can be used if the total cost of electricity must not exceed $50.00 per month. This problem is made more meaningful when students are assisted in obtaining realistic estimates of the amount of hours required for operating such items as water heaters, refrigerators, and electric heaters.

A more extensive project would be to have students determine the total consumption of electricity in their homes and to propose ways in which the consumption could be reduced by a stated percent. A detailed description of such a project is provided by Post (1975) in an article in the Arithmetic Teacher.

Probability (pp. 170-173)

Consider the following questions: If we surveyed 600 people, how many of them would we expect to have a birthday in March? How many of them would we expect to have a birthday on the 20th day of a month?

How many hand-held calculators are there per household in the United States? Discuss techniques that could be used to estimate the answer to this question. Question: How can you sample households so that your sample is representative of the entire United States population?

Suppose that a town has a population of 240,000. In an election for governor, a survey of 400 people showed that, out of 16 people, 10 favored Mr. Dutton, 4 favored Mr. Harrel, and 2 favored Mr. Eggton.

a. If the election were held and all 240,000 people voted, about how many votes would each candidate get?

b. When the election was actually held, the results were:

Candidate	Votes
Mr. Dutton	135,000
Mr. Harrel	77,500
Mr. Eggton	27,500

How do the results compare with the survey? Out of every 16 votes, how many did each candidate get?

To win the world series in baseball, a team has to win 4 out of 7 games. Assuming they are equally matched, what is the probability that a team will win in four straight games?

What is the probability that a family with 3 children has all boys?

(Meiring, Steven.) Problem Solving: A Basic Mathematics Skill. Columbus: Ohio Department of Education, 1979 (to be published).

One of the primary objectives for problem solving during the intermediate and junior high grades should be to provide a variety of problem solving situations for study. Some of the kinds of situations which should receive emphasis include:

- Problems with superfluous or insufficient information.
- Problems that arise from the student's environment or interests.
- Problems which do not have well-defined answers or which involve inexact skills such as estimation.
- Problems posed in nonverbal settings utilizing manipulatives within the classroom or activities outside the classroom.
- Problems developed around central themes which are extended to substantive results but are subdivided into manageable parts.
- Problems requiring students to make choices about the degree of accuracy required or to be used within the problem.
- Multistep problems in which identification of intermediate step meanings are emphasized.
- Problems involving logic, reasoning, testing of conjectures, reasonability of information.
- Problems which require decision-making to result from the mathematical outcomes.
- Problems which emphasize the practical application of mathematics to business or consumer situations.
- Problems requiring the collection and interpretation of data.
- Open-ended and investigatory problems.
- Problems involving modeling to explore interdisciplinary topics.
- Problems which require students to conceptualize and make meaningful very large or very small numbers.
- Problems applying mathematics to school, home, or personal situations.
- Problem situations calling for more than one method of solution.

1. To protest the cafeteria food, Grumpy Smythe decided to send a note to the principal. On the next day, he would encourage three friends to do the same. On the following day, each of them was to find three additional new people to send notes, and so on. How many protest notes will the principal receive on the tenth day? How many notes in all will he have received by the end of the tenth day?

2. A discus thrower has 8 discuses. Seven of them weigh the same amount, but one is known to be slightly heavier. Using an equal arm balance, how can the discus thrower find the heavy discus in just two weighings?

3. A jet plane takes off and flies at an angle of 30° until it reaches a cruising altitude of 10 000 meters. How far will it fly before it reaches this height?

4. Mr. and Mrs. Jogger like to run. From past experience, they know that Mr. Jogger can run six km in the time it takes Mrs. Jogger to cover four km. They have decided that they would like to run the same distance, nine km, and finish together. How much of a head start does Mrs. Jogger need for them to finish together?

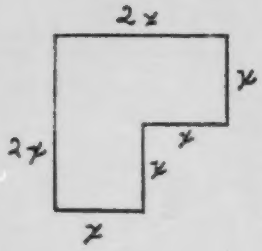
5. At a certain party, there were 36 handshakes exchanged. Everyone shook hands with everyone else exactly once. How many people attended the party?

6. Bread is sliced from a 34-cm loaf into equal slices 2 cm thick. The bread is then toasted for a large breakfast requiring 180 pieces of toast But one of every 5 pieces of toast is too well-done for use. How many loaves of bread are needed?

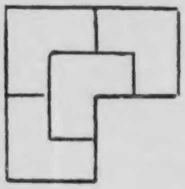
7. A speedboat costs $3500 new. The salesman says that it will depreciate in value by 10% each year. How many years must pass until it is worth half its original value?

8. A man wants to cross a 100-mile side desert by walking. He can walk 20 miles each day. He can carry enough food and water with him for 2 days. The water and food are packaged in one-day amounts for convenience in leaving caches. How many days will it take him to cross the desert and how far will he have walked in all?

9. A farmer with four sons owned property in the shape shown to the right. When he died, his will declared that his land should be divided among his sons so that each would receive the same amount of land and that each son's land should be the same shape. How can this be done?



Answers

1. 19 683 notes on tenth day
   24 524 notes in all
2. Divide the discuses into 3 piles of 3. Compare the two piles. If they balance, the third pile contains the heavy discus. If not, the heavier side contains the heavy discus.
   Divide the heavy pile into individual discuses. Compare two on the balance. The heavy discus will now be known.
3. 20 000 meters
4. 3 kilometers
5. 9 people
6. The slice from either end cannot be used for toast. So only 15 slices per loaf can be used. Of these, 3 will get too well-done. Therefore 15 loaves are required to produce 180 slices of toast.
7. Between the sixth and seventh year, it will become worth half the original value.
8. 15 days to travel 300 miles
9. 
10. Between 10 and 11 rounds
11. 18 points
12. 60

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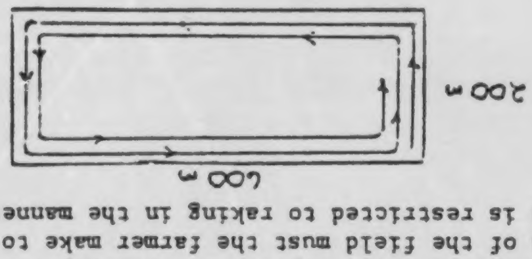
10. Given a hayfield 200 m by 600 m and a hay rake 4 m wide, how many circuits of the field must the farmer make to have raked half the hay? He is restricted to raking in the manner shown.



11. During a game of darts, A can give B 10 points in 50 and B can give C 10 points in 50. How many points in 50 can A give C to make the game fair?

12. How many guests were present as a Chinese party if every two guests shared a dish for rice between them, every three a dish for broth, every four a dish for meat, and there were 65 dishes altogether?

DR. EDMUND EMMER:

THE RESEARCH IN JUNIOR HIGH SCHOOL CLASSROOM MANAGEMENT

The following pages comprise a synthesis of Dr. Emmer's discussion at the R&D Speaks in Junior High School Mathematics seminar. Questions and answers which occurred during the discussion, and hand-outs which were used during the discussion have both been incorporated into the text.

Classroom management refers to the "effective ways in which the teacher arranges time and activities to promote students' contact with curriculum." I am associated with a research project which studies those effective ways. Housed at the R&D Center for Teacher Education at The University of Texas at Austin, the project is called Classroom Organization for Effective Teaching (formerly known as the Correlates of Effective Teaching project) and is directed by Dr. Carolyn Evertson.

Validating good practices

Essentially, COET seeks to validate good practices in classroom management. We do process/product research; that is, we attempt to relate activities to pupil outcomes. We also do experimental research using promising results from the process/product work. Using observational techniques and examining 25-70 teachers per study, we have found that "Good management is predictive of positive student outcomes." Our research, heretofore, has been conducted mainly with five studies listed here in the order that they were carried out:

1. Texas Teacher Effectiveness Study (2nd and 3rd grade levels)
2. Junior High study
3. First Grade Experimental Reading Group Study
4. Third grade, beginning of the year study
5. Junior high, beginning of the year study

For the purposes of this seminar, I will primarily draw on the second and fifth studies. By the way, you may have noted the two beginning-of-the-year studies. We have been investigating the effects of a teacher's management during the first day of school on the operations of the class throughout the rest of the school year. We have actually been in about 80 classrooms at the beginning of school, making observations and collecting data which will be compared with similar observations and data collected later in the year.

A brief historical perspective

In order to demonstrate the evidence which classroom management research indicates, we will use some case studies. We will briefly examine some of the common lines of investigation, accept some common terms and concepts, and then examine two cases in the light of both. Let me first consider the background.

For a good orientation to both the present and the past in classroom management, the best resource is:

- Classroom Management. The Seventy-eighth yearbook of the National Society for the Study of Education, Part II. Edited by Daniel L. Duke. Distributed by the University of Chicago Press, Chicago: 1979.

Especially recommended are the following articles:

- II. Doyle, Walter. "Making Managerial Decisions in Classrooms."
- VI. Brophy, Jere E., and Joyce G. Putnam. "Classroom Management in the Elementary Grades."
- VII. Feldhusen, John. "Problems of Student Behavior in Secondary Schools."
- VIII. Corno, Lyn. "Classroom Instruction and the Matter of Time."

Besides providing you with a good synthesis of the field, these articles will indicate the kind of evidence classroom management research has produced. And, as with every research focus, there have been several approaches to classroom management research. These are the major lines of inquiry in classroom management literature--they indicate ways of observing and even influencing classroom management.

- Teacher Directiveness.
  Some of this literature is useful because it is descriptive of what kinds of behaviors occur in classrooms, but it has not led to anything definitive regarding suitable management strategies.

- Teacher Characteristics.
  The attempt to determine effective teacher characteristics has not been a very fruitful line of research. It has indicated some things, however, such as "good classroom managers have a sense of purpose, a strong idea of what to do."

- Behavior Modification.
  Very specific; induces short term changes but not necessarily stable over time.

- Communication Skills Approaches: (e.g., Reality Therapy, Teacher Effectiveness Training).
  Uses theories on helping relationships, etc. Used frequently, but not much evidence of effectiveness available or attempted.

- Group Management Approaches.
  Line of inquiry exemplified by J.S. Kounin's work. Essentially he studied how teachers manage behavior in groups. Ability to do more than one thing at a time to keep activities flowing smoothly and to be aware of behavior in all areas of the classroom are several variables he associated with freedom from deviance.

- Student Engagement.
  Examines the time that the student is attentive and how the teacher maximizes it. Student engaged time is considered as an important dependent variable.

Probably you will find, whether you read the Classroom Management publication or investigate one of the above lines of inquiry further, that there are several terms used whose meaning may seem self-evident, but for the sake of clarity, let's look at their definitions as used in our research.

Classroom management terms and concepts

These are concepts and terms used in the junior high classroom organization study at COET. You will find them useful in reviewing research findings in most classroom management topics.

CONCEPTS AND TERMS USED IN THE JUNIOR HIGH CLASSROOM ORGANIZATION STUDY

On-task and off-task behavior. On-task is another way of saying "doing what you are supposed to be doing"; off-task is another way of saying "doing what you are not supposed to be doing" or "not doing what you are supposed to be doing." For example, if a student is working quietly on an assignment or turning in an assignment at the teacher's desk, that student is on-task. On the other hand, if students are talking while the teacher is explaining an assignment, or if the teacher has instructed the students to take out paper and pencils, and students are reading books, they are off-task.

Transitions. Transitions are intervals of time between academic activities in which the primary activity is moving from one thing to another. This may include actual physical movement of students or it may be a matter of replacing some materials and getting out something else. The length and efficiency of transitions are factors affecting the smooth functioning of a class.

Procedures. Procedures include *routines* established by the teacher to facilitate functioning of the class. A procedure is a set of guidelines about how to do something that has to be done on a regular basis. The most important characteristic of a procedure is that it has been planned and presented to the class as a way of doing something. In classes with simple, clearly defined procedures, there is little confusion and shorter transitions. Procedures may include getting pencils sharpened, paper and books distributed, getting the students' work turned in, etc.

Rules. Rules are similar to procedures in that they are established agreements about behavior in the classroom, but they are more often definitions of what *not* to do. When rules are carefully chosen and communicated by the teacher, students have clear ideas of sanctioned and unsanctioned behavior in that class. Rules may govern when talking is allowed, when it is appropriate and inappropriate to move around the class, chew gum, eat snacks, etc.

Interruptions. In this study, interruptions are defined as distracting events which force the teacher's attention (and often also the class's attention) to some unplanned event. This might include a messenger coming into the room from the office, an announcement over the loudspeaker, a fire drill, a fight in the room between two students, or a student getting sick and being sent to the office. The teacher's manner of responding to and coping with interruptions is an important aspect of classroom management.

Dead Time. This is an interval of time in which the student or students apparently have nothing that they are supposed to be doing. They are either between activities, or have been left temporarily by the teacher with no provision made for their becoming involved in an instructional activity.

Signals. A teacher may use a variety of signals to get students' attention, to tell the class to get quiet, or convey other messages. In secondary school, a teacher may signal by maintaining silence, standing at a certain place in the room, snapping her fingers, or making statements such as "All eyes on the board." The teacher may also use posture, movement, or eye contact as signals. Signals are most effective when they are consistent and deliberately used. Often the teacher announces important signals to students at the beginning of the year.

Expectations. This word is becoming increasingly common in psychological jargon. The teacher communicates expectations of student behavior in various ways. Explicitly, a teacher tells them what to do and what not to do. Implicitly, a teacher communicates expectations by saying one thing and doing another (e.g., the teacher tells students to stop talking. They continue. After three times, the teacher gives up. The students learn that if they persist, they can have their way. They learn that the teacher's expectations are not really for them to cease talking.)

Credibility. This term follows expectations. Credibility is established when the teacher consistently enforces rules, demands students' compliance with directions, and follows through. with stated penalties for infractions. When the teacher says, "Stop that or I'm going to keep you all after school," can he or she do it (Can kids who ride the bus be kept after school?) and will he or she do it? (Or is it a bluff?) If a teacher has high credibility, students believe what he or she says.

Accountability. The teacher's credibility is also affected by the extent to which students are held accountable for the work that is assigned. Teachers establish high levels of student accountability in several ways: by consistently checking and/or grading classwork and homework; by setting standards for neatness and completeness; by enforcing due dates; by keeping parents well-informed of students' progress; by communicating clear objectives and goals to students; and by helping students keep records of their own work, grades, and progress in the course.

Task Orientation. In task-oriented classes, students show willingness to do the work assigned. They usually understand the objectives, which have been clearly described by the teacher. Students' and teacher's activities are clearly related to academic goals. In classes with poor task orientation, there is much off-task dawdling and socializing. Assignments are not taken very seriously by students or the teacher, and students are often not held accountable for their work.

Testing of Limits. A student tests a teacher by getting up to go to the pencil sharpener just after the teacher has said, "Everyone in their seats." Different students test for different reasons. Some do so to get attention, some to get information about teacher's expectations. Others might do it because they are unclear about where they stand in relation to the teacher.

Critical Incident. This is an occasion in which the teacher is challenged or looked to for a decision which will set a precedent for future similar occurrences. An example of this is when a student fails to hand in an assignment. What the teacher says or does, the degree of confidence with which it is said or done, and the effect on the other students all make up parts of a critical incident.

Monitoring. Monitoring means maintaining surveillance, being the hawk-eye. A good monitor knows as much as possible about what is happening in the classroom at all times. A teacher monitors students' work by circulating around the room, checking papers, redirecting the misguided, encouraging appropriate behavior.

Traffic. Classrooms have traffic patterns just as cities do. The location of the pencil sharpener, the box for turning in homework, and the bookshelves are areas of the room which must be considered in room arrangement. A bit of forethought and planning can prevent these places from becoming competitors with the teacher. Desks should be arranged to avoid crowding and to give the teacher easy access and a clear view of every student's desk.

Feedback. At the beginning of school, the classroom is literally a place where strangers are getting to know each other. The teacher is the most influential single individual in that group. Consciously and unconsciously, he or she gives constant feedback concerning behaviors. Expectations are communicated with feedback. When a student attempts to call out an answer instead of raising a hand, the teacher's feedback to that behavior ("Please raise your hand," or ignoring students who are calling out and recognizing those raising their hands) will establish a precedent for similar situations in the future. Feedback can also be instructional. Notes and grades on papers which have been handed in serve as feedback, as do compliments which the teacher gives a student who has answered a question correctly or brought up an interesting point.

The scenarios

Now, keeping in mind the possible lines of inquiry in classroom management and the concepts and terms just reviewed, let's look at two case studies, taken from two different teachers' classes. In one class, the levels of student achievement (adjusted for entering achievement) and student attitudes were high. In the other class, they were low.

Note: During the seminar, participants read both cases, discussed which teacher was the better manager and why. This discussion is not recorded here in order to invite the reader to review the cases and make his/her own analysis.

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Implications of COET classroom management research

Let's compare our conclusions regarding the scenarios to the conclusions of COET studies that have been done.

- Direct instruction
  Increasing evidence is accumulating supporting direct instruction in classrooms. Direct instruction places the teacher as the decision-maker, with an emphasis in cognitive areas. Direct instruction involves small groups or the whole group but highly individualized, self-paced activities are de-emphasized. Both teacher directed work and seatwork are staple ingredients.

- Classroom rules
  Effective managers give careful thought to classroom rules; in fact, they teach the rules. During the first several days of school, they include clear presentations and discussions of rules and procedures. These rules are the subject of reminders and review during the year. and are the object of insistent enforcement.

- Planning
  Effective managers demonstrate meticulous planning. They incorporate a variety of activities and materials, and conduct discussions with the class as a whole.

- The first week
  During the first week of school, the effective teacher stays in charge. Initially, there is only whole class treatment using simple activities needing little or no explanation.

- Focus
  The effective teacher quickly establishes a content focus and an accountability system for teacher and student. The teacher has high expectations and communicates them, along with procedures, systems, and content to be mastered, to students.

- Directions
  Clear directions are given by the teacher who does not "assume too much" about the students' understandings and experiences.

- Personal information
  Effective teachers seem to solicit personal information from the students at the beginning of the year to give recognition to their individuality. Likewise, the teacher offers personal information of his/her own.

If you are interested in further study in this area, I recommend:

Evertson, C.M., Anderson, C.W., Anderson, L.M. and Brophy, J.E. "Relationships between Classroom Behaviors and Student Outcomes in Junior High Mathematics and English Classes." *American Educational Research Journal* (in press) [This should be available this year. For a report on which the article is based, write the first author, at the R&D Center for Teacher Education, U.T., Austin, Texas 78712.]

Evertson, C.M., Emmer, E.T. and Brophy, J.E. "Predictors of Effective Teaching in Junior High School Mathematics." *Journal for Research in Mathematics Education* (in press--it should be out soon) [For a report, write the R&D Center, Report No. 4069.]

To obtain a copy of the Center catalogue, listing reports that may be obtained for a small charge, write The Research and Development Center for Teacher Education, The University of Texas at Austin, Ed Annex, Austin, Texas 78712.

RESEARCHERS ON THE FIRING LINE

A. Mathematics Questions/Answers
   Dr. Marilyn Suydam

B. Classroom Management Questions/Answers
   Dr. Edmund Emmer

The following are the written questions which the participants presented to the researchers, and the answers and recommended resources with which the researchers responded.

(Suydam)

Replies to Specific (Written) Questions

Problem Solving

[See also: CEMREL/RDIS bulletin: "Mathematical Problem Solving: Not Just a Matter of Words".]

1. Comment on techniques in problem solving as they relate to in-service for mathematics teachers.

   I am enclosing two sections of the packet of materials we put together for the Ohio Regional Conferences [Suydam, M. N.; Cummins, K.; Gibney, T.; Hill, J.; Meiring, S.; and Pikaart, L. Ohio Regional Conferences on Mathematics Education. Columbus, Ohio: The Ohio State University, 1978. ERIC: ED 164 328]. We discussed the key ideas in these materials with the teachers in the workshop-type conferences. We then involved them in solving problems, providing a variety of problems that could be solved in a variety of ways.

   An important stage is to find out what teachers want in in-service sessions, and work from there. To help teachers to change the way they teach problem solving can be handled through heightening their awareness of the need (e.g., by citing assessment data) and through specific help on what to teach and how to teach it. Some effective in-service procedures are described in:

   a. Osborne, A. (editor). An In-Service Handbook for Mathematics Education. Reston, Virginia: National Council of Teachers of Mathematics, 1977.

   b. Didactics and Mathematics. Palo Alto: Creative Publications, 1978.

   Several sources of problems which are interesting to teachers *and* appropriate for middle-school students are:

a. Mathematics Resource Project

These materials, developed at the University of Oregon with support from a grant by the National Science Foundation, are now commercially available from Creative Publications (P. O. Box 10328, Palo Alto, California 94303). The materials consist of five kits, each containing worksheets, calculator activities, games, puzzles, bulletin board suggestions, project ideas, and teaching didactics. The resources were designed for use in grades 5 through 9. The activities are highly motivational and designed to provide practice with all the basic skills including problem solving, mental computation, estimation, and measurement. Individual kits are organized under the following titles:

Number Sense and Arithmetic Skills (832 pages)

Ratio, Proportion and Scaling (516 pages)

Geometry and Visualization (830 pages)

Mathematics in Science and Society (464 pages)

Statistics and Information Organization (850 pages)

b. Mathematics Problem Solving Project (MPSP)

These materials were developed with support from a grant by the National Science Foundation to a project located at Indiana University, the University of Northern Iowa, and the Oakland (Michigan) Public Schools. A number of publications in ERIC describe the project and summarize data from it; the following documents contain the actual sets of problems:

SE 026 919	Organizing Lists
SE 026 220	Using Lists
SE 026 922	Using Guesses to Solve Problems
SE 026 923	Using Guesses
SE 026 925	Using Tables to Solve Problems
SE 026 926	Using Tables

The documents should be available from EDRS shortly (they were not announced in the July 1979 issue of RIE, but may be in the August or September issue, not yet received).

For background information on problem solving, the following sources are suggested:

a. Goldin, Gerald A. and McClintock, Edwin. Task Variables in Mathematical Problem Solving. Columbus, Ohio: ERIC/SMEAC. To be available late Fall 1979.

b. Publications from the New York State Department of Education:

SE 027 565 Creative Problem Solving
SE 027 566 Problem Solving (Revised Edition)

c. (Meiring, Steven.) Problem Solving: A Basic Mathematics Skill. Columbus: Ohio Department of Education. To be published.

This two-volume publication is designed to provide "units" for in-service study, covering such topics as types of strategies and classroom management for problem solving, plus sets of problems for students in grades 3-9. Teachers are given practice in applying the strategies to problems as they read/work the text materials.

d. Krulik, Stephen. 1980 NCTM Yearbook: Problem Solving. Reston, Virginia: National Council of Teachers of Mathematics. To be published April 1980.

e. Greenes, Carole; Gregory, John; and Seymour, Dale. Successful Problem Solving Techniques. Palo Alto: Creative Publications, 1977.

f. Greenes, Carole E.; Willcutt, Robert E.; and Spikell, Mark A. Problem Solving in the Mathematics Laboratory: How To Do It. Boston: Prindle, Weber & Schmidt, 1972.

2. What kinds of problem solving with mathematics are most appropriate for junior high students?

Junior high students need to learn to use problem-solving strategies, just as all students (and as all of us do). The specific strategies they are taught will depend on the kind of problem-solving program they have had in the elementary school. If they have been taught only a general problem-solving strategy, then they should be taught, and given practice in using, the variety of specific strategies in junior high school mathematics. If they have been taught specific strategies in elementary school, then they should practice these, and add to their repertoire any they've missed. Looking back strategies may be particularly useful to the students, to get them involved in thinking about and talking about how problems are solved. They key thing is to give time to problem solving: time to solve problems, time to talk about how they have solved problems. Few present mathematics programs allow sufficient time for problem solving: it is treated in too routine and constricted a fashion, with the focus on getting the answer rather than on how one goes about getting an answer.

Suydam et al. ERIC: ED 164 328

AN OVERVIEW OF PROBLEM SOLVING IN ELEMENTARY SCHOOL MATHEMATICS

Ohio Regional Conference on Mathematics Education

"Learning to solve problems is the principal reason for studying mathematics." (National Council of Supervisors of Mathematics (1978), p. 147). Knowledge, skills, and understandings are important elements of mathematical learning but it is in problem solving that the student synthesizes these components for the purpose of achieving a goal, answering a question, or reaching a decision.

Though most educators agree on the importance of problem solving, nearly any discussion about its role in the elementary school curriculum raises issues in need of further clarification. Four questions which usually emerge are: (1) What is the relationship of problem solving to basic skills? (2) Exactly what is meant by problem solving? (3) How should problem solving be taught? (4) Whose responsibility is it to help students develop problem solving abilities? That is, when should problem solving be introduced in the curriculum?

The purpose of this paper is to present brief answers to these four questions and thus orient you to the problem solving approach taken in these materials.

Is Problem Solving a Basic Skill?

Computational proficiency has been long recognized as a basic skill goal of the elementary mathematics curriculum. The role and importance of problem solving in the elementary program has been generally less well defined. Traditionally, problem solving has been pursued with less immediacy and emphasis than computational skill development and, consequently, has not been considered by many as a basic goal. However, current interest in redefining learning outcomes considered as basic--that is, learning areas important for mastery by all individuals--suggest the need for an expanded definition of basic skills.

Assessment results have highlighted a fact known informally to us all: computational skill proficiency is not synonymous with the ability to apply these skills in meaningful situations. Basic skills by today's standards are judged in terms of competencies. That is, we are concerned with not only what an individual knows but with how well he can use this knowledge. Problem solving constitutes this additional dimension of the basic skill definition in the area of mathematics.

"I think problem solving is the basic skill in mathematics. By problem solving I mean more than knowing what to do, in the sense of having knowledge. I mean having a kind of commitment to problem solving--a willingness to tackle a problem even when one doesn't know right away what to do, and to keep plugging away at the problem until one finds a reasonable solution." This quote comes not from a mathematics educator but from a Georgia state legislator.

While it may be surprising to some to think of problem solving as a basic skill, its development marks the difference between mathematically literate and minimally functioning individuals. A learner with uncultivated problem solving capabilities is a passive and limited individual dependent upon others for his growth and needs. But a learner with well developed problem solving skills is an active, confident, and flexible individual capable of reshaping his learning environment and satisfying his own needs. Education can no longer afford a narrow definition of basic skills. Development of active learners must be the first priority. Consequently, problem solving must be considered a basic mathematical skill.

What is a Problem?

The word problem is derived from the Greek problema, which translated literally means "something thrown forward" (from ballein, "to throw"). Less inspiring but more common is the standard definition, "a question raised for inquiry, consideration or solution . . . a source of perplexity . . ." (Webster's New Collegiate Dictionary, 1975). For our uses then, a problem is a perplexing question or situation. It is important to note that a problem is not simply a question or situation--it must be perplexing.

A question or a situation can be judged perplexing, and thus be a problem, only in relation to a person and a time. What is a problem for one student now may not be a problem for that student in another month or year, or it may not be a problem for another student now. Hence teachers must select questions and situations which will likely be problems for their students.

Another implication of our definition is the idea that a question or situation must be accepted by the student as a problem. "Perplexing" implies that the question or situation is of some interest and that the student will accept it.

The characteristics of a problem for a student, then, are that:

- It is a question or a situation.
- It is accepted by the student.
- At the time it is presented to the student, there is some blockage or challenge so that the solution is not immediate.

Problem solving is the ability to solve problems. We are interested in problem solving in mathematics--the ability to solve problems which use mathematics. However, ultimately problem solving for tomorrow's adults is an important facet of life--be it mathematical or not. Questions such as, "Should I buy a new car?" or "How can I finish painting this chair?" may or may not be mathematical, but they can be real problems to people. Our aim is to equip students with problem solving strategies and our hope is that these students, tomorrow's adults, will be able to use these strategies to solve the problems they must encounter.

How to Teach Problem Solving

In the past, problem solving has been most often taught by presenting specific *types* of problems. Textbooks are arranged so that every so often there are one or two pages of "word problems," all very similar in setting and in solution technique. In upper elementary, junior high, and secondary school you find distance-rate-time problems, work problems, mixture problems, or age problems. In elementary school problem types are often associated with mathematical skills--multiplication problems, fraction problems, or merely problems. Actually, such "problems" are really *exercises* to practice very specific skills. Results are that the student must continually practice the skills to maintain the ability and that he or she is equipped to solve only those problems which are very similar to the exercises.

What we suggest is that the students learn very general *strategies* for attacking problems. (One set of these strategies will be described with examples. of their use in another paper, but for now, three are mentioned to indicate the sort of activity we mean--guess and check, construct a table, or work backwards). There are four important points associated with using a strategies approach to teaching problem solving:

(1) Students can be taught each strategy. The strategies themselves are general skills or abilities.

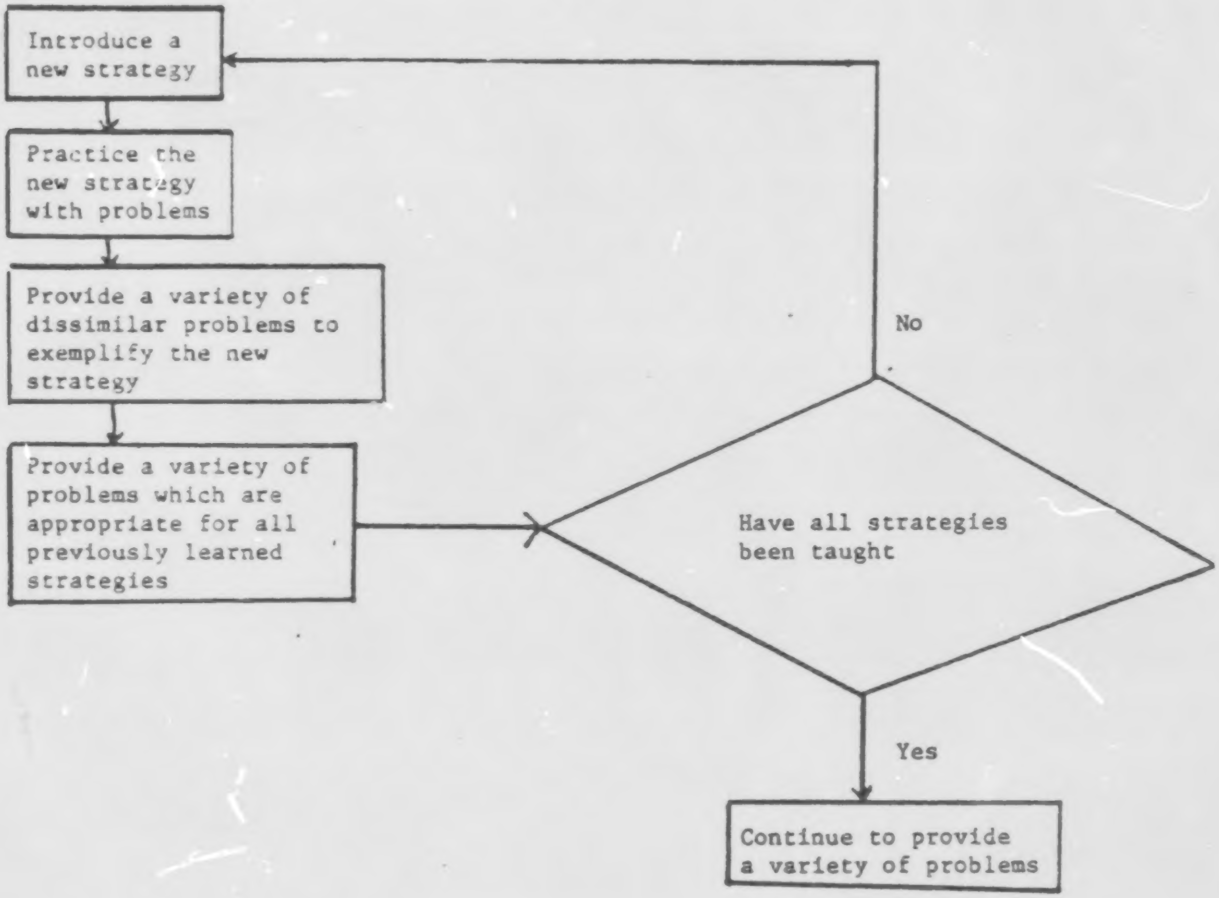
(2) Each strategy can be used in a great variety of problems.

(3) Often a student will use from two to four strategies in a single problem (as opposed to acquiring one technique to solve one type of problem.)

(4) When faced with a new problem a student armed with a set of attack strategies has a way to *begin* to work. (Often the most difficult step in problem solving is getting started).

In general, to teach a strategies approach to problem solving a teacher will follow this procedure:

Teaching Problem Solving


Introduce a new strategy
Practice the new strategy with problems
Provide a variety of dissimilar problems to exemplify the new strategy
Provide a variety of problems which are appropriate for all previously learned strategies
Have all strategies been taught
No
Yes
Continue to provide a variety of problems

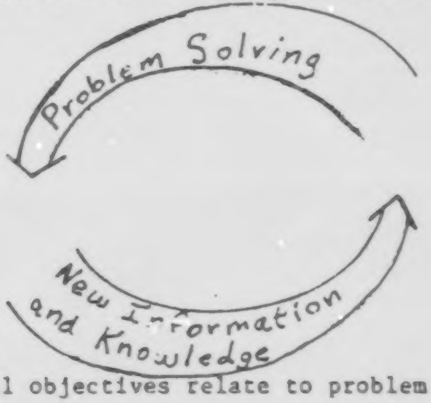
It is clear that to use this approach teachers need (1) a clear idea of all strategies, (2) a large resource of good problems, (3) time in the teaching schedule (mathematics period) to teach problem solving. Most of the strategies appropriate for the elementary program are familiar to teachers and little time is needed to understand them. Developing a resource of good problems is a continuing process. Supplementary materials provide one rich source. Also, teachers can use their textbooks as a resource. But even more fruitful, teachers can make up a great variety of problems themselves (and share them with each other). Finding time for the approach involves, (1) using the current classroom time devoted to teaching problem solving and (2) organizing the learning sequence so that part of the time for practice of computational skills is included in the problem solving activities--during problem solving, students are using computational skills and thus practicing them.

The big pay-off in the strategies approach to problem solving is that the students will experience a variety of problems--the strategies are so general that they can be used in all types of problems. The student acquires experience in solving problems which are new to him or to her. Thus when receiving an entirely new problem, one we cannot even anticipate now, the student has a set of strategies to employ which hold promise of helping to find a solution. In this way, we hope to prepare students to solve problems in the future--even problems we can't think of or know about today.

When Should Problem Solving Instruction Begin?

Problem solving begins as soon as we become aware of our environment, develop some need which we determine to be within our capability of addressing, and go about trying to satisfy this need. Soon after birth, we begin this kind of problem solving and continue it through the rest of our lives.

Insofar as learning is involved, we use new information and knowledge to extend and expand our problem solving capabilities. But we also use problem solving to acquire additional knowledge and skills. Thus, problem solving and learning are inextricably interwoven in the way we naturally discover and react within our environment.



How then do educational objectives relate to problem solving? A broad statement of purpose of the function of education might be: (1) to provide activities that encourage the development of skills and information that are known to be valuable for participation within one's society, and (2) to examine formally processes which are productive for acquiring further information and satisfying one's needs.

This definition brings us back to a comparison between computational skill development and problem solving goals in mathematics education. Quite accurately, computational skills can be judged as fulfilling the first aspect of the function of education and problem solving the second. However, many curriculum designers and other educators use this artificial separation of these two basic goals of mathematics as the basis for curriculum separation and sequential treatment that places computation development before problem solving.

The unfortunate consequences of this decision are threefold. First, the natural manner in which we learn is suddenly interrupted and replaced by a learning model that concentrates on only one aspect of mathematical learning at a time. Secondly, formal consideration of problem solving is withheld from many students until some level of computational proficiency is attained. For some students this means that they will never be exposed to formal problem solving processes. And it means for other students the adjustment to a sharp change in philosophy and emphasis as the focus shifts from computation to problem solving. Thirdly, problem solving as a process too frequently becomes an incidental by-product in problem situations where the primary emphasis continues to dwell on skill refinement and maintenance, application practice, and further computational development. Sadly, this equates to a sizeable number of students never gaining the opportunity to become skillful problem solvers.

To avoid this undesireable and relatively ineffective procedure for learning mathematics, we advocate that the natural mode of learning be continued from nonformal early childhood activities to later formal school learning experiences with an appropriate balance maintained for both aspects of mathematics education. This implies that problem solving should begin in kindergarten at an appropriate level of sophistication. And its development should progress from a natural learning tool to a formal activity in which the focus is on the process as a reproducible strategy that can be utilized to solve similar and dissimilar problems. Continuing responsibility for improving this basic skill is shared by every teacher throughout the rest of the mathematics curriculum. Only with this kind of emphasis can we expect to realize the goal of confident and flexible problem solvers who can assume an active role in modern society.

Suydam et al. ERIC: ED 164 328

PROBLEM SOLVING IN THE CLASSROOM

Ohio Regional Conferences on Mathematics Education

Improving one's effectiveness in teaching problem solving skills requires consideration of many components that affect the success and richness of such learning experiences. Grouping of students, mode of presentation, external and internal variables, realistic modeling, and avoiding pitfalls are some of the important elements that will be discussed in this paper.

Grouping

Three kinds of basic grouping situations can be used effectively in teaching problem solving: large group instruction, small group instruction, and individual learning. Each has advantages, disadvantages, and a role in helping students develop broad problem solving skills.

Large Group. Large group activities are effective for introducing and practicing the application of a new problem solving strategy like "construct a table"; for examining a variety of different methods of solution of the same problem; and for discussing general aspects of problem solving such as initiating strategies and looking back strategies.

Large group instruction has the drawback that individual problem solving growth may be difficult to foster directly for any but the quickest students because individuals respond at different rates to problems and in different ways (favor individual strategies). Therefore, a quick student may solve a problem posed to the whole class before other students have had a chance to consider it thoroughly. Moreover, the wide range of problem solving abilities may make a problem appear trivial to some students and impossibly difficult to others. For these reasons, discussions *about* problem solving are possible with large groups but the *process* of solving problems may be practiced by very few of the group.

Small Group. Small group instruction makes it possible to group students according to problem solving ability and interest. This makes the task of selecting problems of an appropriate degree of difficulty much easier than with large groups. If the group is not too large, students also have the opportunity to engage in group problem solving efforts. In such small groups, students can generally solve more problems than those who work alone, but the groups may take a longer time on each problem than for pupils working alone. Group discussion in order to reach agreement on how to proceed has been shown to produce significantly better achievement than being told how to solve the problem.

Disadvantages of small group problem solving include the need for a large list of problems of varying degrees of difficulty, classroom management procedures, and difficulties in determining individual problem solving growth.

Individual Learning. Problem solving by individuals has the greatest potential for developing problem solving skills that can be both easily ascertained and measured. The individual, left to his own resources, can progress at his rate, use strategies that are comfortable to him, and experience the "aha" feeling wholly on his own. And problems can be selected for individuals according to their needs, interests, and abilities.

Here again, however, monitoring progress and providing individual assistance is limited even more dramatically. Furthermore, one must be more alert to motivation needs, stress concerns, and incorrect procedures than in small or large group efforts.

Mode of Presentation

Real life problems come in a variety of formats and under a mélange of circumstances. Consequently, teaching efforts should be directed toward varying the mode in which problems are presented for solution. Such variety will improve student capabilities to transfer school learned skills to practical problem situations.

Teacher Directed. Teacher led problem solving in which a particular strategy (or computational process) is applied to a group of very similar problems is the most common form of school problem solving activity. Although the customary procedure is to use this approach to practice and refine computational techniques through word problems, variations of this procedure can be used to focus more directly on the problem solving process.

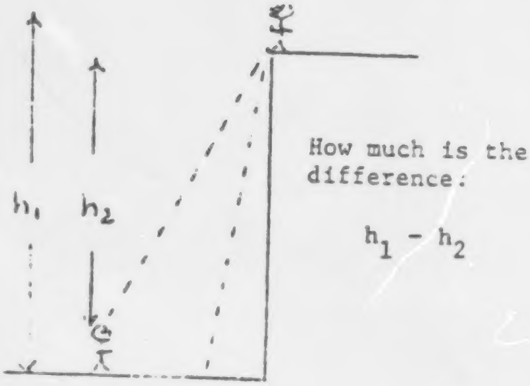
Rather than working problems through to completion, a class discussion of a problem set can be restricted to a thorough talking through of each problem to assure an understanding of the given information, constraints, and problem task. Students can be asked to restate a problem in their own words in order to determine whether they fully comprehend the situation posed by the problem. This activity will help students develop the skill of clear identification of the problem task and help them differentiate between relevant and irrelevant information.

Asking students to describe the problem situation without reference to specific numbers can also help them to generalize the problem situation and processes involved for solution. In this way, student attention will be focused on the problem process, away from the specific answer, enabling an appreciation of the wider application possibilities of the solution process.

Another teacher directed technique with a whole class is to use a problem approach to motivate the need for further mathematical knowledge. By posing a relevant and realistic problem situation that students cannot solve (or solve only with great difficulty), a teacher can motivate the need for new mathematical techniques by answering before it is asked the question, "Why do we need to know this stuff?" And frequently, the new process can be skillfully developed within the context of solving the motivation problem.

Teacher directed problem solving efforts with small groups or individuals call for less direct involvement by the teacher as the problem solving process shifts to the student. After posing a problem of suitable difficulty, the teacher assumes the role of "silent partner," monitoring progress carefully, and reluctantly asking directive questions only as a student exhausts his resources or wears thin his patience. This is the most critical point of the instructional process. Acting too soon will encourage the learner to be dependent upon the teacher. Acting too late will produce anxiety and defeat--the precursors of mathophobia.

Student Generated. A most overlooked opportunity for problem solving instruction arises out of situations that come from students. Using situations that are student generated or revolve around student activities can help bridge the chasm between school practice and actual life applications.



For example, suppose a student observes the phenomenon that standing atop an object and looking down appears greatly different than standing below and looking up at the object. He asks whether there is any physical explanation for this common observation.

A great opportunity for problem solving is at hand! By throwing this question back at the class and asking them what kinds of strategies they might use (listing them for review) to resolve this problem, a very rich albeit unplanned activity can result. Students will see a reason for what they have been learning and discover how to go about applying their skills to a question that intrigues them from real life.

Similar opportunities can be drawn from student activities. Many examples of relevant student situations are found in textbooks. Abstracting these situations from a book format and casting them in a familiar locale with real people will not only enhance the interest level of students. They will also generally find such situations easier to comprehend and be able to transfer the solution process more easily to related problems.

Indirect. Solving a problem is perceived by most students as a challenge. This permits a teacher a third mode for problem solving--indirect presentation. Having established a "place" where interesting and challenging problems are located, students can either elect to use some of their undirected class time to use the "problem place" or be encouraged to that resource by the teacher.

Such "places" might be an attractively decorated bulletin board with The Problem of the Week; Sleuths Corner (or table) where a problem resource is located; stations where problems and equipment or materials useful to solution are gathered; or simply a resource file, cards, or other collection of problems in a selected place (including texts or other resource books.)

An advantage of indirect problem presentation is that a student has flexibility in being able to choose problems which he finds interesting or otherwise appropriate. Another is that he can experience the full problem solving process at his own pace without the threat of failure.

Problems posed for the teacher by the indirect presentation method include establishing a reward (reinforcement) system, student monitoring and evaluation, and matching student ability with problem difficulty. These concerns can be resolved, however, through means such as indexing problems with point values related to their difficulty or use of class problem progress charts that identify the problem, solver, and strategy used.

External and Internal Variables

Associated with problem solving instruction are several factors that affect the learner's ability to solve problems. Some of these factors are external to the learner and are easily controlled through the teaching situation. Others are internal to the learner and are therefore less easily influenced by the teacher.

External variables. Time is a primary consideration in teaching problem solving. The student must have the necessary opportunity to mull a problem over thoroughly: time to understand fully the task and conditions, time for incubation and illumination, and time to think about the solution. An important task for the teacher is to encourage students to extend the amount of time they are willing to work on a problem before "giving up."

All of these factors clearly demand more time from the curriculum; teaching for problem solving is not as efficient as teaching by rote. Therefore, rearrangement of some teaching priorities may be necessary. Or opportunities to teach other subject matter through problem solving may need to be utilized.

Learning style is another important external consideration. Learners respond differently to different stimuli. Some students may need tactile aids to understand the posing of a problem. Others may not. Some students work well independently. Others need more structure and direction. Some students may find certain problem solving strategies more productive than other strategies. Other students may disagree with this list.

Teachers also vary in their uses of instructional styles. There are usually some problem solving approaches that a teacher feels comfortable with and others that he feels less skillful in using. Generally, a teacher is well advised to use methods that utilize one's strengths while giving time to instructional approaches that one feels less expert with. This will widen the range of problem solving experiences for students, capitalize on a teacher's known strong areas, and give the teacher the opportunity to expand his expertise.

Motivation is both an internal and external variable. Most teachers are quite adept at being able to present learning situations in a highly interesting way. But in problem solving the motivation problem centers more around the need for continuing motivation that is sustained through teacher and peer recognition and reinforcement of accomplishments. One goal of problem solving instruction is to encourage students to work more independently and to expand their ability to deal with frustration. Therefore, it becomes very important for the teacher to recognize and reward efforts toward this goal even though a successful problem solution may not be immediately forthcoming.

Internal variables. A problem is not necessarily solved when a correct solution has been found. It is not truly solved until the learner understands precisely what he did and why it was appropriate.

This is one example of what we mean by an internal consideration--a variable that describes the extent of understanding within the learner. Generally we can describe certain elements that are related to this process.

1. Being able to recognize important features of a problem and associating these features with promising solution steps is an important problem solving skill.

2. The process of understanding a problem is related to the ability to restate the problem in an appropriate representation. For example, many problems can be classified as subtraction problems because subtraction is the necessary process required for solution. However, the problems may pose different structures within which that subtraction will occur. Choice of an appropriate numerical sentence to describe the structure can aid in the understanding of the problem. One might choose from the forms

$c - b = ?$ $\qquad$ $c - ? = a$ $\qquad$ or $\qquad$ $a + ? = c$

each of which leads to a subtraction solution.

3. For a learning situation to be appropriate for problem solving development, the techniques necessary for its solution must be well within the student's range of capabilities and experience.

4. Meaningful problem solving experience should not be restricted to repetitious practice with the same technique applied to similar problems. Such experiences should consist of some practice of different approaches applied to similar problems and the same technique used to solve very different problems.

5. Protection from errors is not desireable in problem solving. Students should be encouraged to be sensitive to reasonability of results, to detect errors, and to explain where a mistake lies.

6. Skills and understandings are most useful in problem solving when they have been developed through problem solving.

7. Excluding lack of knowledge and limited mental capacity, there are known factors that cause some problems to be difficult to solve:

a. misleading incorrect solutions - the learner halts his problem solving efforts without realizing his solution is wrong;

b. difficulty in selecting from given alternatives - the learner is not able to systematically reduce the number of possibilities for problem solution;

c. response having low priority in one's range of experience - the learner's background causes him to assign a low probability to productive problem approaches;

d. requirement for generating an unusual response - the problem solution deviates markedly from the solver's past similar experiences;

e. moving too quickly from idea-getting to idea-evaluation - the learner spends too little time generating solution possibilities before trying one;

f. difficulty in identifying surmountable obstacles - the learner fails to distinguish obstacles to problem solution that can be overcome from those that cannot;

g. motivation factors

h. degree of stress

Realistic Modeling

The goal of problem solving instruction is to develop the capability of students to apply successfully school-learned skills to problems arising in real life. However, past experience indicates that transfer of school skills to life situations is not an easy task. The degree to which classroom activities can realistically model real life problem solving is related to how successfully students can make this transfer. Therefore, one of the teacher's tasks is to be aware of realistic problem characteristics.

1. In a real situation, the task to be accomplished or the problem to be resolved is usually well understood. The problem may need to be recast in an appropriate representation, but generally there are no difficulties with interpreting the problem task from given information as is the case in school story problems.

2. Real problems frequently deal with tactile materials and/or real people and situations. Being able to see the problem in terms of concrete materials or to associate it with specific persons or situations makes the problem easier to understand.

3. Real problems have a built-in motivation factor. The successful solution of the problem accomplishes something for the solver. He can immediately appreciate his success or be penalized for his failure until the problem is dealt with. He is therefore willing to wrestle with a problem for a considerable time period. This contrasts to a school situation where a student either moves on from an unsuccessful problem attempt after a frief effort or seeks help from the teacher.

4. There may not be "the" correct solution to a real problem. Many solutions may be acceptable in terms of the specific needs of the solver and the given solution. Rather than requiring an exact answer, it may be sufficient to be "close enough." For this reason, estimation and approximation skills are relatively high in importance in real problem solving.

5. There is no preferred strategy to the successful solution of a real problem. The sole criteria is that the problem be successfully solved in an efficient manner to the problem solver. Therefore, trial and error might be judged just as satisfactory a method as a careful, reasoned problem attack if it solves the problem.

6. Real problem situations frequently reoccur for the solver. This gives the problem solver the motivation to find a solution strategy that will accomplish the repeated task most efficiently. He therefore has a reason to look for more than one solution process to the problem.

7. Real problems are usually cluttered with a lot of irrelevant information or may even be missing some necessary data. One of the solvers first tasks is to distinguish between pertinent and nonpertinent information and to decide whether he has enough information to generate a solution.

8. Few real problems are purely mathematical. The problem may consist of many nonmathematical elements. The task of the solver may be to restate and simplify the problem to mathematical terms. Or a mathematical solution may only be one component to the resolution of a larger problem. Affective considerations frequently play a role in real problem solving.

Obviously, it is not possible nor desireable to simulate each of these real life problem characteristics in classroom problem solving. However, an awareness of them should enable the teacher to occasionally model such situations, making the transfer between school and life skills much easier for students.

Avoiding Pitfalls

In problem solving, as for most areas of instruction, there are a meshing of more than one set of objectives. That is, we pursue problem solving not solely as an end unto itself but in conjunction with other goals and objectives such as refinement and maintenance of computational skills. In trying to accomplish more than one goal through an activity, it is easy to lose sight of the main objective or to occasionally get instructional goals at cross purposes. The following listing contains frequent pitfalls involving problem solving that might arise in this way and that should be avoided.

a. Overemphasis on verbal cues such as equating the word "of" with "times" should be avoided. Such a practice may lead students to misinterpretations for given circumstances as well as focusing attention to early on the parts of the problem before he gains a grasp for the situation wholly.

b. Do not insist that a particular procedure be used in the solution of a set of problems. If the intent is to practice a particular procedure, then the objective may well be computational rather than problem solving.

c. Try not to imply to students that one strategy of many which can be used to solve a problem is the "best." The intent is to increase the number of such strategies that a student has command over. And what may be best for one person may not best for another.

d. In working with large groups, avoid instructional practices that involve a designated learner--that is, a class representative that does the talking, thinking, and actual problem solving for the class with no measure of how many other students understood or participated in what occurred.

e. Consider problem solving as an activity suitable for all students--not just the more capable ones.

f. Problem solving should not be withheld from any group of students until they master a certain set of basics. It can and should be an instrumental activity in the development of basic skills.

In summary, the task of teaching for problem solving development can appear to be an awesome task. But like most complex activities, the goal is well worth pursuing. However, in this case, the task is not as difficult as it might first seem. Most of the components for good problem solving instruction are ingredients for good teaching of any mathematical topic. And many of the aspects discussed in this discourse on problem solving are common tools of the good teacher.

Anxiety

[See also: CEMREL/RDIS bulletin: "Motivation in Mathematics".]

1. Research reports average effects. Since the great proportion of variation in student achievement is predicted by verbal and abstract reasoning, little variation is associated with anxiety. But that variation is averaged over the entire population and therefore appears small by comparison. This does not say that anxiety is unimportant in individual cases. React!

   Of course it's important to take steps to ease anxiety about mathematics for those students who are anxious about mathematics! And the point above is well-taken: the amount of variance attributable to anxiety may be small, but it *is* a problem that we need to be aware of in the case of *many* individuals.

2. Suppose you are working with a junior high student who indicates symptoms of math anxiety. Outline steps you would take to help this student overcome this anxiety.

   a. Talk it over.

   b. Provide experiences the student will enjoy -- but which promote the mathematical idea you want to teach. For example, games like Krypto and Wiff'n Proof provide a lot of practice with computation at various levels. Programming a calculator or computer to solve a problem calls for application of other ideas and skills. Analyzing real problems from newspapers calls attention to the pervading strands of percentage, measurement, and similar problems in everyday life.

   c. Promote the idea that every problem won't be solved instantly -- but, using various strategies, problems can be solved and mathematics can be learned. Encourage students to use common sense in doing mathematics -- then learn standard procedures to shorten the process.

   d. Provide experiences with which the student will be successful: what these are depends on the student.

3. What are ways to deal with math anxiety: are there model examples?

I have been unable to locate specific information on programs at the pre-college level which specially attack anxiety -- although many programs and projects at least purport to attack affective factors. (Note that funding agencies have focused attention on the college student and adult: these are the programs, therefore, that have received the initial publicity. I can believe that there are specific attempts being made to deal with anxiety at the secondary-school level: perhaps soon some of these ideas will be given publication space.)

A course for in-service mathematics teachers was cited in the September 1979 issue of the Arithmetic Teacher. Sponsored by the National Science Foundation, the goal of the course is to strengthen elementary school teachers' understanding and knowledge of mathematics; topics presented include math anxiety, concepts of arithmetic, creative problem solving, uses of calculators, and intuitive geometry and art. (for information, contact Bernard A. Fleghman, Department of Mathematical Sciences, Rensselaer Polytechnic Institute, Troy, New York 12181.)

Diagnosis/Remediation

[See also: CEMREL/RDIS bulletins: "Diagnosis: Taking the Mathematical Pulse" and "Unlocking the Mind of a Child: Teaching for Remediation in Mathematics".]

1. What attention to the use of mathematics in other subjects should be integrated into in-service programs? Can any of these concerns be determined in the process of diagnosis?

   One way to integrate mathematics and other subjects is by selecting and using problems that arise in or across subject areas. The 1979 NCTM Yearbook, Applications in School Mathematics, is one source of such activities. The "challenges" developed by the USMES project (e.g., see ERIC: ED 142 362/388, available on loan from ERIC/SMEAC) also provide interdisciplinary focus.

   The answer to the second question would seem to be "probably". Noting how well a student does with the mathematics that arises in other subject areas may provide some clues to mathematical difficulties or lack of difficulties. Or, diagnosis may indicate that difficulties occur in particular types of mathematical applications from one or another content area.

   One conclusion seems clear: to keep mathematics (or any other subject area) isolated and discrete is doing a disservice to the student. For most people, mathematics appears to be esoteric because we have made so few attempts to relate the ideas of mathematics as they arise in those subject areas to what goes on in the mathematics classroom. This can change . . .

2. How can a diagnostic-prescription management system at the junior high level be incorporated, given a "traditional" classroom?

   I find this a very difficult question to answer in the abstract: how to do it depends on the particular situation:

   a. the attitudes of the teachers

b. the types of students

c. the climate of the school

d. etc., etc.

Some suggestions may be found in the 1977 NCTM Yearbook, *Organizing for Mathematics Instruction*.

The Research Council on Diagnostic and Prescriptive Mathematics is also a source of information. (For publications, contact Don Bernard, 1741 SW 37th Place, Gainesville, Florida 32608; for membership, contact Grace Burton, College of Education, University of North Carolina, Wilmington, North Carolina 28406.)

3. *Would you comment on the potential for diagnosis which future developments in microcomputers present?*

Technology can give us much information on how children learn, and therefore much information pertaining to diagnosis. Unfortunately, this information must be "processed" by the teacher -- and for the teacher to find time to do this is going to be a problem. An example of this problem lies in several of the state assessment programs, where scads of data are output on the individual responses of each child -- teachers indicate that they simply do not have the time to give the data more than a cursory examination; they haven't time to analyze for meaning, nor to plan instruction specific to the needs of *each* individual. One solution is, obviously, to involve the student *much* more directly in analyzing the information so that he or she can use it to develop his or her own individualized program. And computers can be programmed to aid in this process -- for instance, by providing specific hints and clues, by directing students to materials which will help to resolve the problem or learning difficulty, or by providing materials presented in a way which will prevent the problem from arising (given, of course, that we can find how to program the computer to do this). Some persons feel that this is the educational pattern of the future -- the computer managing the learning program in conjunction with each individual child.

Differentiating, rather than individualizing, may be the clue to how to remediate within the total class situation. That is, teachers simply do not have time for, nor can most manage, individual diagnosis/remediation for each student -- but they can provide different activities to meet the needs of groups and subgroups. Thus, for instance, the computer can be used to provide drill-and-practice activities for some students, while providing enrichment via programming for others.

Miscellaneous

1. We need information regarding the content covered within the scope of a junior high curriculum.

   One good source of information lies in curriculum guides; for instance, the Berea (Ohio) School District has developed an excellent guide (which is available from them at a cost of approximately $25). Obviously, textbooks and state curriculum guides also suggest appropriate content -- and how to teach it.

2. I'm concerned about getting current research findings incorporated into the curricula of teacher preparation and administrator training programs in higher education., What suggestions do you have for keeping higher education faculty up-to-date with research and for encouraging their flexibility in coursework to revise and update their work with their classes and as consultants in the field?

   I'm sure my suggestions are those you've already thought of: attending meetings, reading, holding discussion groups, etc., etc. And we know how responsive everyone is to such things! The only real suggestion I have is to "package" ideas in an interesting way: for instance, provide bulletins, syntheses, practical translations — and keep them brief and meaningful. And perhaps provocative, in some cases -- to provoke discussion . . .
   Much more thought obviously needs to be given to this question. , .

3. What are useful sources of information on calculators?

   Some selected references on calculators as we work toward effective use of them as integral components of instruction are:

   Burt, Bruce (editor). Calculators: Readings from the Arithmetic Teacher and the Mathematics Teacher. Reston, Virginia: National Council of Teachers of Mathematics, 1979. [$5.00 to members; $6.25 to non-members]

   The four sections in this just-published collection of articles pertain to policies, uses, and selection of calculators; calculators and the curriculum; calculator activities; and research (only four articles are in this latter section, however).

Humphreys, Casey et al. Calculator Cookery. Minneapolis: Minneapolis Public Schools, East Area Curriculum Office, 1977. ERIC: ED 146 009.

Worksheets were designed for use with low-achieving ninth and tenth graders; they are also appropriate for seventh and eighth graders. Activities are included on: exploring algorithms, pattern searches, estimation and reinforcement of basic computation, consumer applications, and societal applications. Notes for teachers are included with each of the lessons.

Judd, Wallace P. Problem-Solving Kit for Use with a Calculator. Chicago: Science Research Associates.

This kit contains a variety of problems, categorized by type. It is appropriate for use at upper elementary levels into secondary school.

Suydam, Marilyn N. Reference Bulletins from the Calculator Information Center. ERIC: SE 026 880. Information Bulletins from the Calculator Information Center. ERIC: SE 027 798. Calculators: A Categorized Compilation of References. ERIC: SE 027 796.

These materials, developed for and distributed by the Calculator Information Center, list references to instructional applications and research, and provide information on how to use calculators at particular grade levels. Some of these materials are still available from the Center (1200 Chambers Road, Columbus, Ohio 43212). However, the status of the Center is uncertain, pending decisions by the National Institute of Education.

Iowa Problem-Solving Project (ESEA Title IV-C). (Joan Duea, George Immerzeel, Earl Ockenga, John Tarr, and Jack Wilkinson.) Cedar Falls: University of Northern Iowa, 1978.

The purpose of this project is to develop materials to improve the mathematical problem-solving abilities (using calculators) of students in grades 5-8. The materials are available in classroom sets for experimentation: 1 teacher's edition with teacher's notes, record sheets, answers, and tests; 15 pupil copies of skill booklets (approximately 30 pages); and a 100-card problem deck. Modules include: Problem Solving ... Using the calculator, Using Tables, Using Guesses, Using Resources, Using Calculator Codes.

Mathematical Problem Solving Project, University of Northern Iowa, 1975. Calculator Handbook. ERIC: ED 161 758. Getting to Know the Calculator. ERIC: ED 161 757. Problem Solving: Opening the Door Using the Mini-Calculator. ERIC: ED 161 759.

The Handbook presents student worksheets with activities in counting, place value, estimation, the four operations with whole numbers, number patterns, perimeter, inequalities, decimals, decimal equivalents of fractions, and percents, all to be worked with a calculator. The second document provides worksheets to help the child learn how to use the calculator. The last document outlines five different ways in which calculators can be used to develop a student's ability to solve problems: for solving problems which use real-world data, for solving a greater number of standard word problems, for using different approaches to solve problems, for extending problem situations, and for using the calculator in an experimental setting. Each section presents problems which exemplify these five uses of the calculator.

North Carolina. Calculator Inservice Kit. Prepared by Mathematics Division, North Carolin State Department of Public Instruction, 1978.

In this notebook kit are transparency masters, calculator games, calculator activity cards for grades 5 through 9, and calculator problem card decks for teachers. (For further information, contact: George Immerzeel, Iowa Problem-Solving Project, Price Laboratory School, University of Northern Iowa, Cedar Falls, Iowa 50613.)

Report of the Conference on Needed Research and Development on Hand-Held Calculators in School Mathematics. National Institute of Education and National Science Foundation, 1977. ERIC: ED 139 665.

Results of the conference, intended to provide a well-defined framework for future research and development efforts, are reported. Twenty-one recommendations covering the development of an information base, curriculum development, research and evaluation, teacher education, and dissemination are discussed.

Uses of the Calculator in School Mathematics, K-12. Monograph No. 12, Michigan Council of Teachers of Mathematics. March 1977. [Available from MCTM, Box 16124, Lansing, Michigan 48902; price, $1.50.]

Part I considers desirable features of calculators for classroom use, basic assumptions, and curriculum concerns. Part II is a collection of activities for grades K-3, 4-8, and 9-12, with a bibliography.

THE UNIVERSITY OF TEXAS AT AUSTIN
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AUSTIN, TEXAS 78712

*Research and Development Center
for Teacher Education
Education Annex*

September 6, 1979

Ms. Jan Schechter
Southwest Educational
Developmental Laboratory
211 East 7th Street
Austin, Texas 78701

Dear Jan,

Enclosed are some references bearing upon several of the questions which I was asked at the regional exchange meeting. I have dealt with questions in groups and omitted a few which were simply solicitations of opinion without a research base.

Let me know if you need more information.

I certainly enjoyed the conference. Best of luck to you and your husband in Ecuador this year.

Cordially,

Edmund T. Emmer,
Associate Professor
of Educational Psychology

ETE:ara
Enclosure

RECEIVED
SEP 10 1979

(Emmer)

REPLIES TO SPECIFIC WRITTEN QUESTIONS

1. Implications of classroom management research for observation for observation and evaluation of teacher performance.

Borich, G. D. The Appraisal of Teaching. Reading, Massachusetts: Addison-Wesley, 1977.

See Part I, especially.

Dunkin, M. J. & Biddle, B . J. The Study of Teaching. New York: Holt, Rinehart, & Winston, 1974.

See Chapter 6, "Management and Control" and Chapter 11, "Findings for Teachers."

2. What "universal" parameters can be identified that distinguish effective teaching?

Travers, R. M. W. (ed.) Second Handbook of Research on Teaching. Chicago: Rand McNally, 1973.

See Chapter 5, by Rosenshine and Furst, on "The Use of Direct Observation to Study Teaching."

Good, T. & Brophy, J. Looking in Classrooms (2nd ed.) New York: Harper & Row, 1978

Research on teacher effects. Thematic issue of the Journal of Teacher Education, 1976, 27, 5-66.

See also references listed under question 3, below.

3. Can correlational research be used to propose specific behavior changes for teachers or for teacher education?

Brophy, J. E., & Evertson, C. M. Learning from Teaching. Boston: Allyn and Bacon, 1976.

Gage, N. L. Teacher Effectiveness and Teacher Education. Palo Alto, Pacific Books, 1972.

From Borich, G. B. (ed.) The Appraisal of Teaching
Brophy & Evertson (79-95)
Soar (96-103)
Stallings (104-113)
Rosenshine (114-120)
Good & Grouws (121-139)
Berliner & Tikunoff (280-291)

4. Other references, not cited above, relating to classroom management research or applications of such research.

Duke, D. (ed.), Classroom Management. Seventy-eighth NSSE Yearbook (78:2), University of Chicago Press, 1979.

Kounin, J. S. Discipline and Group Management in Classrooms New York: Holt Rinehart, & Winston, 1970.

APPLICATION OF RESEARCH TO INSERVICE

The following are the written recommendations of the participants at the R&D Speaks in Junior High School Mathematics seminar regarding the application of the seminar's content to inservices. Responses were guided by a questionnaire, a sample of which follows the recommendations.

Implications for Inservice

How could the information presented at this conference on the following topics be applied to inservice and to the work you do?

Diagnosis:

- At the Junior High level, it is imperative that we must move in a direction which will equip teachers with diagnostic-prescription related skills. Would take a whole-scale retraining of teachers.
- Most of the work done in these areas seems to be highly individualized. More needs to be said about how to really remediate within a large classroom setting. Diagnosis seems easy but how to remediate in a classroom is not so clear.
- With the mandated minimum standards in math, teachers must know how to diagnose student needs so as to ensure the teaching of those needed skills.
- Train teachers to use SAT scores and other records to identify potential problem areas; instruct teachers how to use practical observation techniques to improve their instructional program.

Remediation:

- Inservice is needed for school administrators as well as math teachers re the necessity for setting up remedial programs utilizing a diagnostic-prescriptive instructional approach.
- Most of the work done in these areas seems to be highly individualized. More needs to be said about how to really remediate within a large classroom setting. Diagnosis seems easy but how to remediate in a classroom is not so clear.
- To assure that students at least master the minimum skills, teachers will be using remediation to a greater extent. Thus, inservice presentations must address ways of accomplishing this.
- Inform teachers about the remediation techniques and suggest practical ways to use these techniques in their classrooms.

Problem Solving:

- Many implications; Must emphasize necessity for development of teachers "problem-solving skills" especially at elementary level. Difference between problem-solving and solving problems should be emphasized.
- Programs will be conducted at local, regional, and state inservice meeting to present this topic. Consideration will be given to a statewide NSF proposal for inservice.

Problem Solving: (continued)

- Teacher involvement in workshop situations is one of the most important aspects in teaching problem solving. Overcoming teacher anxiety in this area is of utmost importance.
- Plan activity-oriented workshops utilizing problem solving techniques.

Math Anxiety:

- Must better establish contact with teacher preparation institutions and the public to develop more positive attitudes toward mathematics.
- The information presented in this area is not definite enough to present inservice. Without specific tasks (programs) an inservice leader might complicate the problem rather than help.
- Teachers need to understand that students can learn to do and enjoy math and that teacher attitude plays a major role in how the students perceive math.
- Do an activity on math anxiety in Teachers Effectiveness Training Workshops.

Classroom Management:

- Topics to be addressed include: 1) Adequate planning, 2) Management by objectives (Evaluation), 3) Teaching/Learning Strategies, 4) Utilization of Learning Materials, 5) Classroom organization.
- More emphasis should be placed on the "effective" procedures outlined at the preservice level. How to communicate this to colleges of education is a needed topic and service.
- Teachers should be aware that the way the classroom is managed probably has more influence as student achievement than other aspects of teaching.
- Teachers will list and discuss positive characteristics of an effective teacher. Taped interviews will be discussed.

To be appropriate for inservice and/or your clients, will you use a different way of presenting the conference information than we used? If so, what formats would be appropriate for which audiences?

- Data (information) gleaned at the SEDL conference will be translated into more usable form for teacher use in the classroom. Teacher inservice will consist primarily of our working with school math staffs and imparting relevant information to be used in a classroom setting.
- Yes, participants need more "hands-on" participant involvement materials.

- In small group inservice, the format used here would be appropriate. In extremely large groups, it becomes important to break them into smaller groups, get them involved and hold them accountable for tasks to be accomplished.

- More time will be allowed for sharing ideas and practical ways of implementing the suggested techniques.

Implications for Dissemination

What are some possible strategies for disseminating this research information in your state through your agency, if relevant, and to your colleagues?

- The Annual Math-Science conference will serve to get much of this information to teachers. The statewide Math and Science Advisory Councils will also be contacted. These efforts compiled with the teacher inservice components of the SDE will serve to get research information to teachers.

- Our state has an active math council. Through the newsletter and statewide conferences, these topics can and will be presented.

- Our SDE has a dissemination department that we will utilize to get the information to the appropriate people. Also, the math section and elementary section publishes a newsletter which will be used for this purpose. State Supervisor holds numerous inservice workshops for the state's teachers.

- State Teachers Organization (magazines and newsletters), Administrators Conferences, State Mathematics Organization (newsletters and conferences), Superintendent's newsletter.

Implications for the Future

Do you have any suggestions regarding future mathematics or classroom management research, or their applications to your state's education priorities in mathematics?

- Future studies on the effect of hand calculators on mathematics instruction, and establishing a diagnostic-prescriptive approach to the teaching of mathematics.

Are there any additional research-related topics in mathematics or classroom management which were not covered in the presentations/discussions which might be addressed in a future conference?

- Minicomputers and mainstreaming

- What effect do minimum competencies have on student achievement in mathematics?

- What are strategies to use with inservice meetings in developing a desire to pursue this topic. Teachers are reluctant to work in a "new" area especially when it is outside the basic text. How to remediate effectively in a classroom with 25-30 students. What research shows on programs for gifted students. What research shows are the best materials and effective utilization of the hand held calculators. How much time and how early (gradewise) should calculators be used? Mini-computer--research and inservice.

- More on Math Anxiety; quality of teaching vs quality of the teacher; effect of hand calculators on math and science instruction; definition or re-definition or math content covered at the Junior High level of instruction.

R3
SEDL
REGIONAL EXCHANGE

R&D SPEAKS IN MATHEMATICS CONFERENCE

August 8-9, 1979

____
STATE

Implications for Inservice

How could the information presented at this conference on the following topics be applied to inservice and to the work you do?

Diagnosis: ____

____

____

____

Remediation: ____

____

____

____

Problem Solving: ____

____

____

____

Math Anxiety: ____

____

____

____

Classroom Management: ____

____

____

____

To be appropriate for inservice and/or your clients, will you use a different way of presenting the conference information than we used? If so, what formats would be appropriate for which audiences?

Implications for Dissemination

What are some possible strategies for disseminating this research information in your state through your agency, if relevant, and to your colleagues?

Implications for the Future

Do you have any suggestions regarding future mathematics or classroom management research, or their applications to your state's education priorities in mathematics?

Are there any additional research-related topics in mathematics or classroom management which were not covered in the presentations/ discussions which might be addressed in a future conference?

EVALUATION SUMMARY

Several sources of data were analyzed to evaluate the August 7-8, 1979 SEDL/RX "R&D Speaks in Math" conference. These include a post-conference participant questionnaire, a post-conference consultant questionnaire, and a number of indirect measures reported by the conference organizer which indicate that participants were active in exchanging useful information.

The objectives of the conference, indicated in a pre-conference letter to participants and consultants, were to provide an opportunity for practitioners and researchers to:

- meet and discuss critical questions in instruction in order to learn from each other, and
- present some ideas and suggestions that would help in teacher inservice.

The available data indicate that the conference was successful in meeting these objectives.

Participants Questionnaire Results

Fifteen of the sixteen conference participants returned the post-conference questionnaire. The participants from the six state SEDL/RX region held the following job titles: math consultant, national program director, classroom teacher, math specialist(s), coordinator of state program validation, associate professor of mathematics, elementary special services in mathematics, math section chief, state math consultant, science and conservation specialist, SDE secondary math specialist, math supervisor, supervisor K-8, and associate professor of math education. Individuals attending the conference answered one open-ended question and rated nine statements on a five-point Likert scale (1=strongly disagree, 2=disagree, 3=neutral, 4=agree, 5=strongly agree.) Participants agreed with all statements, indicating that:

1. There was ample opportunity to react to the information and interact with the consultants and participants (X=4.2).
2. The suggested techniques and ideas were practical (X=4.2).
3. The conference was relevant to my (the practitioner) needs. (X=4.3)
4. The math research will be useful in inservice. (X=4.3)
5. The seminar format contributed to positive learning interactions. (X=4.4)
6. The conference heightened my awareness of research in math and classroom management. (X=4.7)
7. The research on classroom management will be useful in inservice. (X=4.7)
8. The conference was well organized. (X=4.8)
9. It would like a follow-up R&D Speaks in Math in my state. (X=4.9)

The fact that participants agreed most with the final statement regarding a follow-up conference strongly indicates that the conference was perceived as useful and worthy of their time and energy. The similar high mean response obtained for the statement concerning heightened participant awareness of math and classroom management research is a further indication that the conference was a success.

Additional evidence to support this conclusion is found in the responses to the open ended question "As a result of the conference, I will _________." The summary below reveals that participants gained useful and practical information from the conference. According to this question,

- Six individuals will disseminate information from the conference to others in their state via inservice, newsletters, and other activities;

- Two persons will be better prepared to conduct inservice in their states;

- Two participants will share the information on classroom management and math anxiety, feeling more confident of the procedures that are being used in the classroom;

- One individual will incorporate the information into criteria and instruments to validate state math programs;

- One will immediately use the classroom management information in a talk to Independent School District Senior High math teachers;

- Another will be more aware of sources of research and will make teachers more aware;

- One participant intends to work to improve student achievement, teacher competency, and math program development.

- One individual will utilize RX services to get needed information to math teachers.

A suggestion was made by one conference participant that more emphasis be placed on adequate planning to carry out the classroom management research.

Consultant Questionnaire Results

Both consultants at the conference responded to the questionnaire and indicated that:

- The participants at the conference learned new and useful information. One consultant hypothesized that participants may have just heard what they knew expressed in different words.

- As a result of the conference, the consultants gained an increased understanding about the needs of practitioners. One consultant expressed concern regarding the American tendency to search for one simple solution.

- The participants at the conference suggested that research should be undertaken regarding the development of appropriate methods for dissemination of research results to teachers.

- The conference was both well organized and relevant to the needs of the participants.

- Their time was well-utilized.

- The format allowed participants the opportunity to raise questions and discuss relevant issues while providing adequate time for the consultants to present necessary background and information.

Indirect Measures

A number of observations by the conference organizer indicate that the conference was a success and its objectives were met. Some indirect measures of success are as follows:

- The panel discussion was extended forty minutes to allow the consultants to respond to the large quantity of questions.
- The consultants (researchers) indicated they were interested in completing a written report matching questions with available research and resources, an activity not required in their contracts.
- After completion of the presentations, participants remained to ask additional math related questions. Conversations at the hotel in the evening centered around math.

Conclusion

Direct and indirect measures indicate that the "R&D Speaks in Math" conference was valuable to the conference participants and presenters. The involved parties met and discussed critical questions in mathematics instruction and gained ideas that will aid in their conducting inservice training. The following letters from participants exemplify reactions to the conference.

Appalachia
Educational
Laboratory

August 17, 1979

Ms. Jan Schechter
Southwest Educational
  Development Laboratory
211 East Seventh Street
Austin, Texas 78701

Dear Jan:

Sandy and I both want to thank you again for the invitation to attend your R&D Speaks in Mathematics last week. The experience of actually taking part in a conference similar to the one which I am planning was really invaluable--I really got a "feel" for the kind of activities about which I had been verbalizing but not really understanding.

I was glad, too, to have a chance to meet you before you sail away on your adventure. My good luck wishes go with you.

I'm sure I'll talk with you again before you leave.

Again, thanks. I stole some of your ideas!

Sincerely,

Jane

Jane L. Williams
Intern

JLW:ksc

RECEIVED
AUG 20 1979

Appalachia Educational Laboratory, Inc. • Educational Services Division
1031 Quarrier Street/P.O. Box 1348 • Charleston, West Virginia 25325 • (304) 344-8371
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AUG 15 1979
TITLE IV, PART C

August 14, 1979

M E M O R A N D U M

TO: Dolores Dietz

FROM: B. K. Graham and Bill Trujillo

RE: SEDL R & D Speaks on Mathematics Conference

It was our pleasure to attend the conference "R & D Speaks on Mathematics," August 8 and 9, 1979. Our impressions of the conference were very positive, ranging from the consultants to the logistics of the meeting.

As a result of the conference we feel we'll be able to provide those teachers with whom we come in contact with information about math topics currently facing teachers. We plan to use the annual Math/Science Teachers Conference to disseminate some of the information, as well as the Name of Site meeting in Albuquerque in October.

We are grateful to the Lab and its staff for inviting us to the conference.

rg

RECEIVED
AUG 21 1979

CONCLUSION

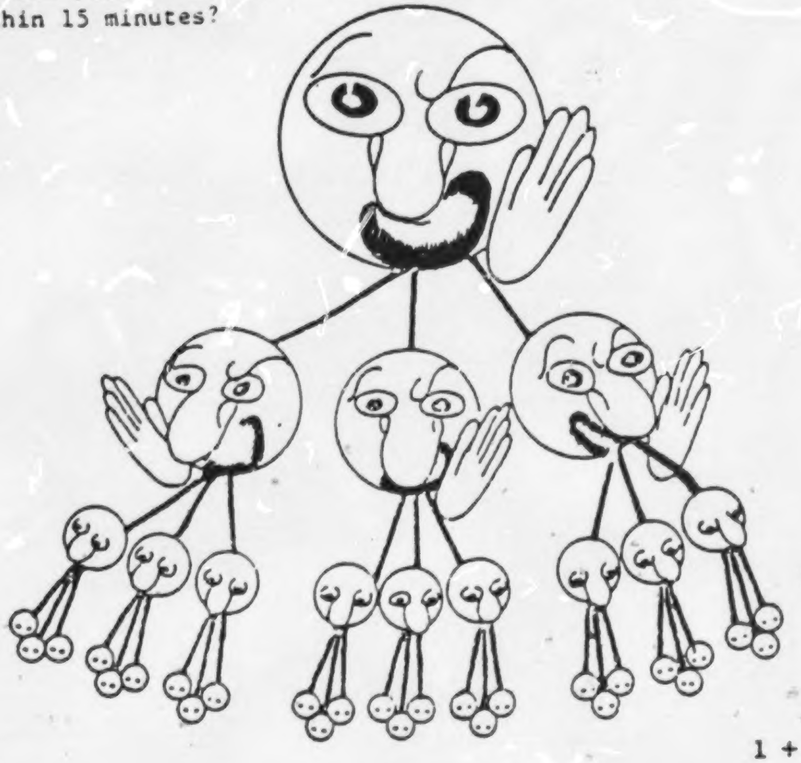
As this report is being completed, one month after the seminar, some use of the information from the seminar has already been indicated by one state education agency and one other regional exchange. A follow-up questionnaire is being included with the September 1979 distribution of this document to the participants, and more information will then be available regarding the activity's effectiveness.

Seminars such as these incorporate a kind of "Selective Dissemination of Information" method. That is, the actual seminar is available to only a few representatives from each state, but if they are "key people," the information will be disseminated to many others in each state through newsletters, inservice, conferences, and even casual conversation. It is an indication of the effectiveness of such an approach that we hope to trace, though we recognize the difficulty of doing so. How many times will the research information be transformed? In how many ways will it be useful? How will it be passed along? We are only beginning to estimate the answers to these questions, but the general dissemination theory at work may be described in a junior high mathematics problem:

RUMORS

OR I HEARD IT THROUGH THE GRAPEVINE

How long would it take to spread a rumor in a town of 80,000 people if each person who hears the rumor tells it to three new people within 15 minutes?



1

8:00
1 person

1 x 3 = 3

8:15
1 + 3 = 4 persons

3 x 3 = 9

8:30
1 + 3 + 9 = 13 persons

9 x 3 = 27

8:45
1 + 3 + 9 + 27 = 40 persons

Hopefully, though, we're not spreading rumors--just valid research findings.

APPENDIX A:

Planning Documents

Southwest Educational Development Laboratory
211 East 7th Street, Austin, Texas 78701

512/476-6861

June 1, 1979

The Southwest Educational Development Laboratory's Regional Exchange Project has been contributing to state education agency dissemination efforts in Arkansas, Louisiana, Mississippi, New Mexico, Oklahoma, and Texas, guided by an Advisory Board member from each of these states. In your state, we have been working with

has recommended that we contact you in connection with a series of seminars we are sponsoring. One of our goals is to highlight those R&D outcomes which might be of direct assistance to teachers. To do this, we are sponsoring seminars in selected content areas (reading, mathematics, special education, multicultural education), but our immediate target audience is not teachers; rather, it is the SEA or ISA subject specialist. We feel that a regional project such as ours can be most effective by supporting SEA specialists who work with teachers during inservice and/or staff development

We call the seminars "R&D Speaks" and we are beginning the planning for "R&D Speaks in Mathematics." An "R&D Speaks in Reading" has already been held.

Our plan is to bring one or two researchers together with you to focus on specific mathematics topics of concern to you and the teachers you work with. We want to emphasize those R&D results, recommendations, and techniques which would be of most use during inservice, staff development or other similar situations.

I have enclosed a list of possible topics for the mathematics seminars. Would you have time to select the five which are of most importance to your work? I would like to call you June 7 or 8 to discuss the topics with you, and to get your reactions to these questions as well:

- What grade level focus would be most appropriate for you?
  - K-6
  - 7-9
  - 10-12
  - other?

- We are hoping for a late August or September seminar date. What dates are best for you?

- Would a product display be helpful to you? If so, what kind of display would be of most benefit?

The Regional Exchange will pay travel and per diem expenses for three people to attend from your state. We expect to meet in Dallas, a central location in our region. As planning is completed we will provide more details on time and specific location.

I look forward to speaking with you, and hope that we can plan a seminar which will be useful to you.

Thank you for your time and consideration.

Sincerely,

Original signed by

Janis O'D. Schechter
Dissemination Specialist

JS/ta

Enclosure

cc: Preston C. Kronkosky
Nancy Baker Jones

POSSIBLE TOPICS FOR SEDL/RX SEMINAR

"R&D Speaks" in Mathematics

1. Diagnosis
2. Remediation
3. Mainstreaming
4. September Deficiencies
5. Motivation
6. Priorities-Curriculum
7. Problem Solving
8. Mastery Learning
9. Meaningfulness
10. The Bridge from Concrete to Abstract
11. The Flexible Use of Textbooks
12. Algorithms
13. Role of the Calculator
14. What to do in Kindergarten
15. Metrication-Measurement
16. Mental Arithmetic
17. Drill/Related Topics
18. Grouping
19. Individualization
20. Manipulatives
21. Testing
22. Counting Strategies
23. Professional Development
24. Math Anxiety

APPENDIX B:

RDIS Mathematics Portfolio Bibliography

AEL Rx Resource and Referral Center

Appalachia Educational Laboratory
P. O. Box 1348
Charleston, West Virginia 25325
304/344-8371

RDIS MATHEMATICS PORTFOLIO BIBLIOGRAPHY

Books

Callahan, Leroy G., and Vincent J. Glennon. Elementary School Mathematics: A Guide to Current Research. 4th edition. Washington, D.C.: Association for Supervision and Curriculum Development, 1975, 1952. 188 pp.

The content of this monograph covers the essential aspects of the school mathematics program of the elementary school (K-4 or K-6) and the middle school (grades 5-7). The content will be useful for teachers and supervisory personnel of junior and senior high students since large numbers of these students still experience difficulty in the mastery of elementary and middle school level mathematics.

AEL-Rx 000-925 *ED 116 966* *MF available*

Carpenter, Thomas, and others. Results from the First Mathematics Assessment of the National Assessment of Educational Progress. Reston, Virginia: National Council of Teachers of Mathematics, 1978. 140 pp.

This monograph is a comprehensive presentation and discussion of the 1972-73 National Assessment Mathematics data. It is a report for the NCTM Project for Interpretive Reports on National Assessment. This monograph presents an overview of the whole mathematics assessment, examining data from all four age groups and for all content areas.

AEL-Rx 000-931 *ED 156 443* *Not available from EDRS*

Conference Board of the Mathematical Sciences. Overview and Analysis of School Mathematics Grades K-12. Reston, Virginia: National Council of Teachers of Mathematics, 1975. 155 pp.

This book provides an overview and analysis of the U.S. school-level mathematical education--its objectives, current practices, and attainments.

AEL-Rx 000-932 *ED 115 512* *MF available*

Downes, John P., and others. 76 Questions: A Synthesis of the Research on Teaching and Learning Mathematics. Arlington, Virginia: ERIC Document Reproduction Service, 1977. 119 pp.

The document is a summary of research and recommendations concerning questions frequently asked by teachers and administrators about the learning and teaching of mathematics. The intent is to provide a concise summary of major studies having direct implications for planning and teaching mathematics.

AEL-Rx 000-922 *ED 162 896* *MF available*

Driscoll, Mark J. Annotated School Mathematics: Annotated Bibliography. St. Louis, Missouri: CEMREL, Inc., Research and Development Interpretation Service (RDIS), n.d. n.p.

This annotated bibliography is the result of a two year process by the staff of the Research and Development Interpretation Service (RDIS) to collect research synthesis in elementary school mathematics.

AEL-Rx 000-924 *Not in ERIC*

7/79

Lachat, Mary Ann, and others. Math Programs that Work: A National Survey. 3rd edition. Rochelle Park, New Jersey: Capla Associates, 1977. 52 pp.

A catalog and description of mathematics programs that are working throughout the country.

AEL-Rx 000-938 *ED 156 432* *MF available*

McKillip, William D., and others. Mathematics Instruction in the Elementary Grades. Morristown, New Jersey: Silver Burdett Company, 1978. 374 pp.

A textbook for teachers of elementary mathematics. The techniques and materials were developed specifically to help teachers improve mathematics teaching.

AEL-Rx 000-937 *Not in ERIC*

National Council of Teachers of Mathematics. The Slow Learner in Mathematics, Thirty-fifth Yearbook of the National Council of Teachers of Mathematics (Reston, Virginia: National Council of Teachers of Mathematics, 1972), 524 pp.

The main purpose of this yearbook is to provide ideas for teaching the slow learner in mathematics. It is not an attempt to prescribe specific content in mathematics for all slow learners. The emphasis on objectives in the book is on how to make them clear to students and how to state them in terms of desired student behaviors.

AEL-Rx 000-933 *ED 071 870* *MF available*

Payne, Joseph N. Mathematics Learning in Early Childhood, Thirty-seventh Yearbook of the National Council of Teachers of Mathematics (Reston, Virginia: National Council of Teachers of Mathematics, 1975) 297 pp.

This yearbook is for teachers of young children, designed to help them in making important decisions about teaching and about curriculum.

AEL-Rx 000-935 *ED 107 534* *MF available*

Reys, Robert E., and Thomas R. Post. The Mathematics Laboratory: Theory to Practice. Boston: Prindle, Weber, and Schmidt, Inc., 1973. 297 pp.

The purpose of this text is to explain how the theories of laboratory techniques can be implemented and to assist teachers in initiating a laboratory approach.

AEL-Rx 000-936 *Not in ERIC*

Suydam, Marilyn N. Developing Computational Skills, 1978 Yearbook. Reston, Virginia: National Council of Teachers of Mathematics, 1978. 245 pp.

This yearbook provides a broad expectrum of computational topics. The first article considers the role of computational skills, several others focus attention to helping children master basic facts. Algorithms for operations with whole and fractional numbers is covered as well as suggestions for instructional strategies.

AEL-Rx 000-934 *ED 156 442* *Not available from EDRS*

Books (Continued)

Suydam, Marilyn N. Evaluation in the Mathematics Classroom: From What and Why to How and Where. Columbus: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1974. 26 pp.

This document is intended as a quick reference guide on the topic of evaluation of mathematics instruction.

AEL-Rx 000-595 *ED 086 517* *MF available*

________, and Jon L. Higgins. Activity-Based Learning in Elementary School Mathematics: Recommendations from Research. Columbus: ERIC Center for Science, Mathematics, and Environmental Education, 1977. 97 pp.

The primary objective of this review has been to identify the most viable dimensions or facets of activity-based instruction, especially as they might apply to children needing compensatory education.

AEL-Rx 000-921 *ED 144 840* *MF available*

________, and Donald J. Dessart. Classroom Ideas from Research on Computational Skills. Reston, Virginia: National Council of Teachers of Mathematics, 1976. 58 pp.

This booklet seeks to aid the teacher gain a better understanding of how to help children learn computational skills with meaning. An attempt was made by the authors to summarize selected research results on computational skills with whole and rational numbers under the broad categories of introducing, reinforcing, maintaining, transferring, and applying.

AEL-Rx 000-926 *ED 146 006* *Not available from EDRS*

________, and Fred J. Weaver. Using Research: A Key to Elementary School Mathematics. Revision. Columbus: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1975. n.p.

This booklet consists of eleven bulletins which present answers to some questions about research on elementary school mathematics. These are revisions of the bulletins which were originally prepared in 1970 as one facet of the "Interpretive Study of Research and Development in Elementary School Mathematics". The bulletins are organized by topics, and specific research findings are cited, with lists of selected references included.

AEL-Rx 000-930 *ED 120 013* *MF available*

Weaver, Fred J., and Marilyn N. Suydam. Meaningful Instruction in Mathematics Education. Columbus: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1972. 69 pp.

The focus of this monograph is upon the "other revolution" which was closely associated with the so-called meaning theory of arithmetic.

AEL-Rx 000-705 *ED 068 329* *MF available*

Periodicals

The Arithmetic Teacher, Vol. 23, No. 7 (1976), pp. 492-580.
This issue of the Arithmetic Teacher is devoted to the utilization of hand-held calculators in the classroom.
AEL-Rx 000-927 *Not in ERIC/Available from NCTM @ $2.50*

The Arithmetic Teacher, Vol. 25, No. 2 (1977), p. 60.
This issue of the Arithmetic Teacher is devoted to the subject of problem solving as a basic skill in mathematics.
AEL-Rx 000-928 *Not in ERIC/Available from NCTM @ $2.50*

The Mathematics Teacher, Vol. 71, No. 2 (1978), p. 160.
This issue of the Mathematics Teacher presents a variety of manuscripts that describe and address concerns related to testing for minimum competency.
AEL-Rx 000-929 *Not in ERIC/Available from NCTM @ $2.50*

"Notes from National Assessment: Six Selections from the Arithmetic Teacher," Arithmetic Teacher, December 1975 (pp. 653-657); January 1976 (pp. 62-66); February 1976 (pp. 137-142); March 1976 (pp. 217-220); April 1976 (pp. 296-302); May 1976 (pp. 389-393).
These six selections cover the areas of Estimation, Word Problems-Subtraction; Recognizing and Naming Solids; Processes Used on Computational Exercises; and Addition and Multiplication with Fractions.
AEL-Rx 000-923 *Not in ERIC*

Other

Far West Laboratory for Educational Research and Development. Mathematics Curriculum. Washington, D.C.: Association for Supervision and Curriculum Development, n.d. Chart.
All the mathematics programs included in this chart have been approved by the Joint Dissemination Review Panel (JDRP), Education Division, Department of Health, Education, and Welfare on the basics of positive data. Details on evaluation results can be obtained from both USOE's catalog, Educational Programs That Work, and the sites.
AEL-Rx 000-976 *Not in ERIC*

KEY TO DOCUMENT AVAILABILITY

AEL-Rx 000-000 - *Rx Resource and Referral Center (RRC) Accession Number*
ED 000 000 - *Available from ERIC Document Reproduction Service*
MF available - *Microfilm available*

PUBLISHERS ADDRESSES

Association for Supervision and
Curriculum Development
1701 K Street, NW, Suite 1100
Washington, D.C. 20006

Capla Associates
18 Overlook Avenue
Rochelle Park, New Jersey 07662

Mark J. Driscoll
Research and Development
Interpretation Service
CEMREL, Inc.
3120 59th Street
St. Louis, Missouri 63130

ERIC Center for Science, Mathematics,
and Environmental Education
College of Education
Ohio State University
1200 Chambers Road
3rd floor
Columbus, Ohio 43212

ERIC Document Reproduction Service
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Arlington, Virginia 22210

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Ohio State University
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ERIC Information Analysis Center for
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Columbus, Ohio 43210

National Council of Teachers of
Mathematics (NCTM)
1906 Association Drive
Reston, Virginia 22091

Prindle, Weber, and Schmidt, Inc.
20 Newbury Street
Boston, Massachusetts 02116

Research and Development
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CEMREL, Inc.
3120 59th Street
St. Louis, Missouri 63139

Silver Burdett Company
250 James Street
Morristown, New Jersey 07960

This Bibliography was prepared by Jacqueline Wagner and the AEL-Rx Resource and Referral Center staff from the materials collected by the Research and Development Interpretation Service (RDIS) for the Mathematics Portfolio.

7/79